THINKING MATHEMATICS IN THE EARLY GRADES

REPORT

Research of the achievements at the end of the first cycle of the realization of the project

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Skopje, December 2012

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ОБРАЗОВАНИЕ

CONTENTS

EXECUTIVE SUMMARY

The objective of the investigation

This investigation has been realized after three years from the beginning of the implementation of the project *Thinking Mathematics* with an aim *to collect information about the progress and the effects of using the principles and techniques in the teaching of Mathematics from the lower primary teachers and the achievements of the students at the beginning of Grade Four.*

More specificly, the investigation has been effectuated after one cycle of implementing the project:

1. to collect information about **the changes at the trained teachers** related to their attitudes towards teaching and learning Mathematics, as well as about the change in the level of mathematical and pedagogical knowledge which are promoted in *Thinkong Mathematics,*and are relevant for the implementation of the instruction in Mathematics;
2. to collect information about **the differences in the achievements** at the end of Grade Three of those students taught by the trained teachers in the Project *Thinking Mathematics,* related to the achievements of the students taught by the teachers that have not been trained;
3. to collect information about the implementation of the activities in the project schools, the concerns of the teachers, and the support they are receiving.

Conducting the investigation

Data were collected on a sample of 14 schools, taking part in the *Project Thinking Mathematics* and of 14 schools with similar characteristics (the control schools) that were not included in the Project. In each of those schools, 10 teachers from grade teaching phase and 20 students from Grade Four classes were examined, so that the total number of examinees was 276 teachers and 557 students. In these schools, a baseline study was conducted at the very beginning of the Project. Such an approach enabled reliability in comparing the states in the project schools and in the control schools after three years, since the students in the course of the first cycle were taught by teachers that were trained at the very beginning of the Project. Though the schools are a representative sample of the project schools in phase 1, nevertheless they are not a representative of all primary schools and the data may not be generalized for all the schools.

Data were collected using the following instruments developed for this investigation:

- Scale of attitudes on learning and teaching Mathematics;
- Test on pedagogical knowledge of teachers (related to the Ten Principles of *Thinking Mathematics*);
- Test on mathematical knowledge of teachers;
- Scale of the teachers concerns about the application of the approaches in *Thinking Mathematics*;
- Questionnaire for teachers;
- Test in Mathematics for students with tasks from the contents, studied until Grade III;
- Protocol for the interview with school principals and with pedagogues/ psychologists.

The data have been processed according to established indicators for all examined subjects and comparisons are given of the results between the two measurements in the project schools and in the control schools.

Basic findings of the investigation

A brief description of each indicator and the main findings related to it, for this investigation, is given below (in more detailes the indicators are given in chapter 3 of this Report).

It is stated, within each indicator, and for almost each category examinees, that:

- **There is a significat improvement in the results of the examinees from the project schools measured in 2012, compared to those measured in 2009, and**
- **There is statistically significant difference of the results between the examinees in the project schools and in the control schools, measured in 2012.**

This denotes that the **Project has had positive effects on the teaching of Mathematics in the first cycle of Primary Education**.

Indicator	Description	Findings
Undestanding of learning and of teaching Mathematics	Attitudes of teachers about learning and about teaching Mathematics which are based on the Ten Principle.	The teachers in the project schools, to a considerably higher level than the teachers in the control schools, have changed their attitudes toward the teaching and learning of Mathematics in direction of departing from the traditional approaches and accepting instruction based on the 10 Principles promoted in Thinking Mathematics. The difference in the arithmetic mean in the project schools, between the measuring done in 2012 and the measuring in 2009, is statisitically significant (in 2012 it was 93,14 while in 2009 it was 87,34).
	Pedagogical knowledge of teachers related to the approaches promoted by the Project.	Pedagogical knowledge of teachers in the project schools has improved, compared to that from the study in 2009. In the measuring done in 2012, the awerage result on test is 40%, and in 2009 it was 33%.
	Teachers expectations from their students, related to the achievements in Mathematics.	In comparison to the findings in 2009, there has been a considerable increase of the importance that the teachers in project schools give to all of the curriculum objectives, that they were asked about. 67% of the teachers in the project and in the control schools generally do not expect that their students would achieve more than that prescribed by the curriculum. The state was similar to that with the measuring in 2009. The percent of teachers in the project schools who consider that they have methodical freedom in carrying out the teaching is 58%, and there is no change compared to that from the measuring in 2009.
	Familiarity with Mathematics curricula for the subsequent education cycles.	In comparison to 2009, many more teachers (especially from the project schools) have stated that to a larger extend they are familiar with the Mathematics' curricula for the next education cycles, and particularly for the second cycle of Primary Education.

Indicator	Description	Findings
Teachers' knowledge of Mathematics	The knowledge of teachers and their understanding of the concepts of number, operations and characteristics, doing test tasks and problems' solving.	In the project schools, the results obtained in 2012, from all the test areas, are higher compared to the results in 2009 (especially in doing textual tests and problem solving situations where there is an inprovement of 21 percentage points[1]). The teachers in project schools in 2012, achieved a 8% / 8percent points higher average result on the test in Mathematics' knowledge than the teachers in the control schools. In 2009 there was no statistically significant difference between the results of the teachers in the project schools compared with the teachers in control schools.
Support to changes in the teaching of Mathematics in the project schools	Satisfaction with students' achievements.	In comparison to the measuring in 2009, there has been an increase of the school management staff (more than 2/3 of those interviewed) who has been satisfied with the achievement in Mathematics in the grade teaching phase.
	Considerations about the project Thinking Mathematics.	More than half of the school principals and the pedagogues/psychologists are satisfied with the implementation of the Project, because it has induced positive changes with students.
	Activities in giving support and improving the teaching of Mathematics.	The cooperation among the grade teachers and between the grade teachers and the subject teachers during the implementation of the project activities, have increased. Almost all the school principals and pedagogues/psychologists listed a lot of ways in which they have been included in support to the Project implementation .
	Equipment of schools for the teaching of Mathematics.	The equipment of classrooms with manipulative aids, compared to that in 2009, has been improved, but not so much as to support to an appropriate extent the teaching of Mathematics.

1 Percentage point is the difference between the two results that were expressed as percentages. In this report, for easier reading the text differences are expressed with a numerical value and the % sign. In other words everuwhere where comparioson is mentioned, these #% are percentage points and mostly relate to differences between rezults in the two measurements (2009 and 2012), or between different groups of respondents.

Indicator	Description	Findings
Achidevements of students	Achievements of students on the test in Mathematics which contained tasks that measure conceptual and procedural knowledge, understanding and applying numbers, operations and their properties, as well as doing textual tasks and problems' solving.	The students in the project schools in 2012 have higher average results for 22 percent points, compared to those in 2009. This improvement is alike with the students in the instruction in Macedonian language and with the students in the instruction in Albanian language. There has been a statistically significant improvement in the achievement of students in all teaching areas. The largest one (for 26 percent points large average rezult of the tests compared to that one in 2009) is on the tasks in the area Operations and the properties of operations.
Acceptance, implementation and the need to give support to the implementation of the project Thinking Mathematics	Accepting the project activities.	More than 2/3 of teachers have overcome the concern related to accepting and including into the teaching, the priciples and techiques promoted by the Thinking Mathematics. More than half of the teachers are still concerned about how the implementation of the new methods in the teaching would influence upon the learning and the achievements of students.
	Level of use of the project activities.	The larger number of teachers estimate themselves that they are at the average levels of use of the 10 principles and the techniques in the teaching (mechanical use, routine use and refinement) and this most oftenly happens at more than 1/3 of the teaching hours.
	The need to give support to the trained teachers in different fields.	About ¼ of the teachers, to a great extent, need support in planning the realization of the instruction by including the principles and techniques of Thinking Mathematics. Almost half of the teachers, to a great extent, need support in assessing their students, and one third of them need support in measuring the effects of the implementation of Thinking Mathematics in the early grades.

Recommendations

Taking into consideration the findings presented above, as it was expected - the implementation of the project *Thinking Mathematics* accompanied by a continuing support to the teachers, has improuved the students achievements, as well as teachers pedagogical and mathematical knowledge.

The authors of this Report, on the bases of a detailed analysis of the results and of the factors influencing upon the results, have considered to give the following recommendations to the Project implementers, for the parts in which it is necessary and possible to have even greater improvements.

1. To continue with further realization of the well designed and elaborated training program , the models for support and the certification of the trained teacher.
2. To provide a more strengthened direct, or in other way, systematic designed support (for example by advisors from the Bureau for Develpment of Education, by master trainers, teachers mentors, through learning teams, through organized sharing of good practices and other materials) of the trained teachers to:
   - overcoming the methodical ambivalence and increasing their self-confidance to make use of the methods' diversity and freedom;
   - strengthening the pedagogical and mathematics knowledge and skills, specially related to the assessment of students and undertaking responsibility about the achievements of students;
   - overcoming the concern about the effects of the implementation of the project activities (and in particular the concern about the *consequences, colaboration and refocusing)* with which they would come to a higher level of implementation of the approaches, principles and techniques promoted in *Thinking Mathematics*.
3. The difference between the trained teachers that carry out the teaching in Macedonian language of instruction and in Albanian language of instruction, shows that a more strengthened support is needed for the teachers doing the iinstruction in Albanian language. The Project implementers should develop a specific and more intensive approach.
4. The school management is satisfied with the implementation and with the effects in the project schools, but such considerations are not based on assessments and comparisons of the achievements of students. It is necessary, in schools to introduce a "culture" and practice of systematic observations, especially when long-term teaching innovations that lead to higher results and achievements of students are in plase.
5. The feeling of the teachers, that they have partial support from the school in introducing changes in the teaching, should be overcome by a much bigger understanding of the changes and by including all subjects in the school in introducing the methodological changes.

6. Although the students in the project schools show a considerable improvement in achievements, there are still some curricula contents (for example. assessing, mathematical modeling, understanding of concepts etc.) where the teachers should pay greater attention, which would later provide for students an easier acquiring of knowledge and skills from other mathematical areas. Also, the students achievements should be improved to correspond to the requirements/expectations prescribed by curricula at the end of the first cycle iof Primary Education.

INTRODUCTION

In well-planned action projects, especially in those of wider extent, it is expected, prior the beginning of the project activities, to conduct a baseline investigation, and on a certain key period of the implementation of the project to do another investigation in order to foresee the progress and the effects of the project activities.

This Report presents the findings of the study of the state of affairs at the end of the first cycle[2] of the implementation of the Project *Thinking Mathematics in the early grades* (in 2012) and the comparisons with the findings of the investigation of the beginning state prior to the project activities (conducted in 2009). The Bureau for Development of Education with the support of UNICEF Office – Skopje is implementing this project, and for their needs the research study was conducted by the Macedonian Civic Education Center.

The conceptual framework of the study is set up on the basis of the analysis of factors which could influence upon the effects of the project activities and which could be found in studies of the factors for effective teaching of Mathematics, as well as of the other factors related to the achievement of students in Mathematics. Hence, this investigation is an attempt to provide answers to the following questions:

- What are the experiences and the opinions of the teachers about the teaching of Mathematics?
- What are the teachers' perceptions concerning the importance of certain contents and mathematical skills and what are the expected results?
- What is the level of pedagogical and mathematical knowledge of teachers related to the teaching that are needed for carrying out the teaching of Mathematics?
- What is the level of the support to teachers by the school management in promoting the teaching of Mathematics, and particularly in the project *Thinking Mathematics in the early grades*?
- What is the level of knowledge and understanding of students on items from the areas that are encompassed within the Project *Thinking Mathematics*?

In this Report, the states between the sampled project schools and the control schools are compared on the basis of the results from the measuring done in 2012, and also comparisons of the project schools results are made prior to the implementation of *Thinking Mathematics* (in 2009), and this one (after three years of the Project implementation).

In the first part of the Report, are given detail results from the measuring in order to serve for further planning and implementation of the project activities, as well

2 Under the term first cycle we understand the three years implementation of the teaching of Mathematics in the grade teaching phase in those schools that took part in the activiities (the schools from phase I).

as for subsequent evaluations. The second and the third part of the Report contain informations about the Project and about the methodology of the investigation. The structure of this Report differs from the structure of the *Report of the Baseline Study* (of 2009), in so that, first are presented the informations and the analyses related to the results of the study which the authors considered to be the most important.

PART I – FINDINGS OF THE STUDY

In this part, are given the data obtained by the investigation according to the defined indicators:

- **teachers understanding of learning and teaching, including the pedagogical knowledge of teachers;**
- **mathematical knowledge of teachers;**
- **support to the teaching at the school level;**
- **students achievements;**
- **accepting, implementation and needs for further support to the implementation of *Thinking Mathematis*.**

The data represent an overview of the state of the relevant factors in monitoring the effects in the project schools and in the control schools prior to the beginning of the Project (in 2009 година) and at the end of the realization of the first cycle (in 2012). Generally, the states between the project schools and the control schools have been compared, which are an indicator of the changes resulted by the implementation of the Project. In situations where there has been a need to understand and describe the states better, the changes have been analysed within the project and within the control schools.

With regard to the limitations of the sample, the generalization of the conclusions about the effect of the Project upon the schools with different characteristics should be done very carefully.

1. TEACHERS UNDERSTANDING OF LEARNING AND OF TEACHING MATHEMATICS

The understanding of how students learn Mathematics and how the teacher teaches Mathematics, was examined by using three instruments. Two of them were used in the both measurings (in 2009 and in 2012), while the attitudes of the teachers to Mathematics and to the teaching of Mathematics were examined only in the measuring of the beginning state (in 2009).

In 2009, very high values on the scale of attitudes towards Mathematics and to the teaching of Mathematics were obtained, which means that the teachers have shown themselves in a very positive light. There were no statistically significant differences in the attitudes of the teachers from the project and from the control schools. According to the theory and to the investigations, it is considered that the attitudes to Mathematics are not easily changeable with adults who have already had experience in learning and teaching Mathematics (for ex.. Block & Hazelip, 1995; Kagan, 1992, according to Novotna and all (2006)). Because of the stated reasons, we considered that the attitudes to Mathematics have not changed, and in the repeated measuring we did not use the *Scale of the attitude to Mathematics and to teaching of Mathematics*.

For each of the instruments, below is given a short description and the results from the two measurements as well as the comparisons between them are shown.

1.1. Attitudes of teachers to learning Mathematics and to the teaching based on the Ten Principles

WAY OF MEASURING

The attitudes of teachers were measured on a Linkert's scale which consisted of 30 statements:

- 15 related to learning Mathematics and
- 15 related to teaching Mathematics.

Statements are formulated in a way to express an attitide in accordance with the 10 principles of *Thinking Mathematics* or an attitude contrary to the 10 principles.

The examinees were asked to denote the level of agreement with each statement at a five level scale (from 1 – I do not agree at all, and up to 5 – I agree completely). In the further text this instrument will be named as *Scale of Attitudes to Thinking Mathematics (SATM)*

The results between the project and the control schools are shown with differences in the arithmetical means of the level of accepting the statements on the scale (SATM). Comparisons were made only to the project schools (comparisons to the control schools were made when it was considered that such information would explain the situation deeply). Separately commented are the statements in which the differences between the project schools and the control schools are most obvious.

In the table given below, are presented the results of measuring the attidudes of teachers from the project schools and from the control schools in 2009 and in 2012 (shown through arithmetical means (AM) on the scale SATM), and the comparisons between them.

Table1. *Comparisons of the means on the Scale of attitudes to Thinking Mathematics*

Time of investigation	Project schools	Significance of differences	Control schools
2012	**AM =93,14**	⟷ difference significant at level 0,01	**AM=90,30**
Significance of differences	↕ difference significant at level 0,01		↕ difference significant at level 0,01
2009	**AM=87,34**	⟷ there is no difference	**AM=86,36**

- After three years of implementing *Thinking Mathematics,* the teachers from the project schools, to a higher degree than the teachers from the control schools, accept learning and teaching Mathematics in accordance with the 10 principles, as well as the application of the promoted approaches and techniques to the teaching (AM on the scale for the project schools is 93,14; and AM for the control schools is 90,30 – the difference is statistically significant). In the baseline investigation of the Project, the teachers from the project schools and those from the control schools did not differ on the level of accepting the statements on the *Scale of attitudes to Thinking Mathematics*.
- During the first three years, the attitudes of teachers, in accord with the approaches promoted in the Program *Thinking* Mathematics, became considerably more positive. The change is more explicit with the teachers from the project schools, as it could have been expected. The change of the

attitudes of the teachers from the control schools is due to their participation in other programmes that promote similar approaches. Namely, 23% of the teachers from the control schools, responded that in the last three years they had taken part in training or in projects for promoting the instruction of Mathematics. The most mentioned had been the trainings within the Primary Education Project, which was implemented by USAID support in 2006 till 2011.

- Generally speaking, in the both investigations, the teachers were inclined to accept learning and teaching Mathematics in accordance with the *Thinking Mathematics* aproaches.The average value on the scale, with which the attitude toward such instruction was measured, is considerably above the theoretical average value (AM = 75 in both investigations), with both teachers from the project schools and from the control schools. Certain inclination to the positive part of the scale is expected due to the fact that the pedagogiical knowledge of teachers, and especially that acquired by the complementary training, would enable them to know which approaches are professionally more acceptable. Also there is a tendency of our teachers to show themselves in a positive light[3].

The arithmetical mean with the teachers from the project schools, for 26 out of 30 statements on the scale SATM, is increased compared to that in 2009. It means that the number of teachers, that understand the learning and accept the instruction in accordance with the principles of *Thinking Mathematics,* has increased. The statements, in whose there are biggest changes in acceptance (0.4 or more units on a five level scale) are given bellow.

Mostly reduced is the number of teachers that have agreed with the following statements (which represent the traditional approaches in teaching):

- The ability to memorize formulae and procedures is of key importance for Mathematics.
- In the teaching, I do not have sufficient time to be able to give students various tasks that would induce the understanding of numbers and mathematical operations.
- At my Mathematics' classes it is not allowed students to talk with each other.
- The use of fingers after Grade I, delays the learning of operations.
- Mathematics is in fact a sequence of rules that should be followed.

The greaest differences (0.4 or more units on a five level scale) among the teachers from the project schools and the teachers from the control schools are found in the acceptance of the first four of the statements stated above, as well as in the following statements:

- The group work is not adequate for Mathematics' classes.

3 Almost all the studies (for ex. TIMSS, PIRLS, The National Assessment) support the expressed inclination of the teachers in former Yugoslav Republic of Macedonia to give socially desirable rasponses.

- With small children, it is not to be insisted to do tasks in many different ways, it is sufficient that they know one way of doing them.

These statements represent also the traditional teaching, which to a great extent is accepted by the teachers from the control schools.

Aside from the considerable decrease in the level of acceptance of the listed statements which show acceptance of the traditional teaching, still, there is a great number of teachers from the project schools who also agree to a big extent with the statements that show inclination to accepting the traditional approaces in teaching. The most distinct are the following:

- I must cosistently to realize the curricula.
- The students should first practice the operations with numbers, and then give them do the textual tasks.
- When I am introducing new concept I start from the beginning because of those children that have no previous knowledge.
- The role of the teacher is to transfer knowledge to students.

All these attitudes show to which directions we should work in the further professional development, in the part of changing the attitudes to teaching and learning Mathematics, primarily to: providing a feeling of greater freedom in the implementation of the curricula; increasing the active role of students; paying respect to their previous experience; and a better ballance between the conceptual and the procedural knowledge.

CONCLUSION

- During the three years' implementation of the project activities, the teachers from the project school, to a considerable higher level than the teachers from the control schools, have changed their attitudes to the teaching and learning Mathematics in direction to departing from the traditional approaches and accepting teaching based on the 10 principles promoted by *Thinking Mathematics*.
- Although, generally speaking, the teachers in the projects schools and those in the control schools agree to a higher degree with the statements which describe the teaching according *Thinking Mathematics*, there are some traditional approaches which, to a considerable extent, are accepted by the teachers in the project schools. This points out to their methodical ambivalency (simultaneous acceptance of some traditional approaches and of some innovative approaches).

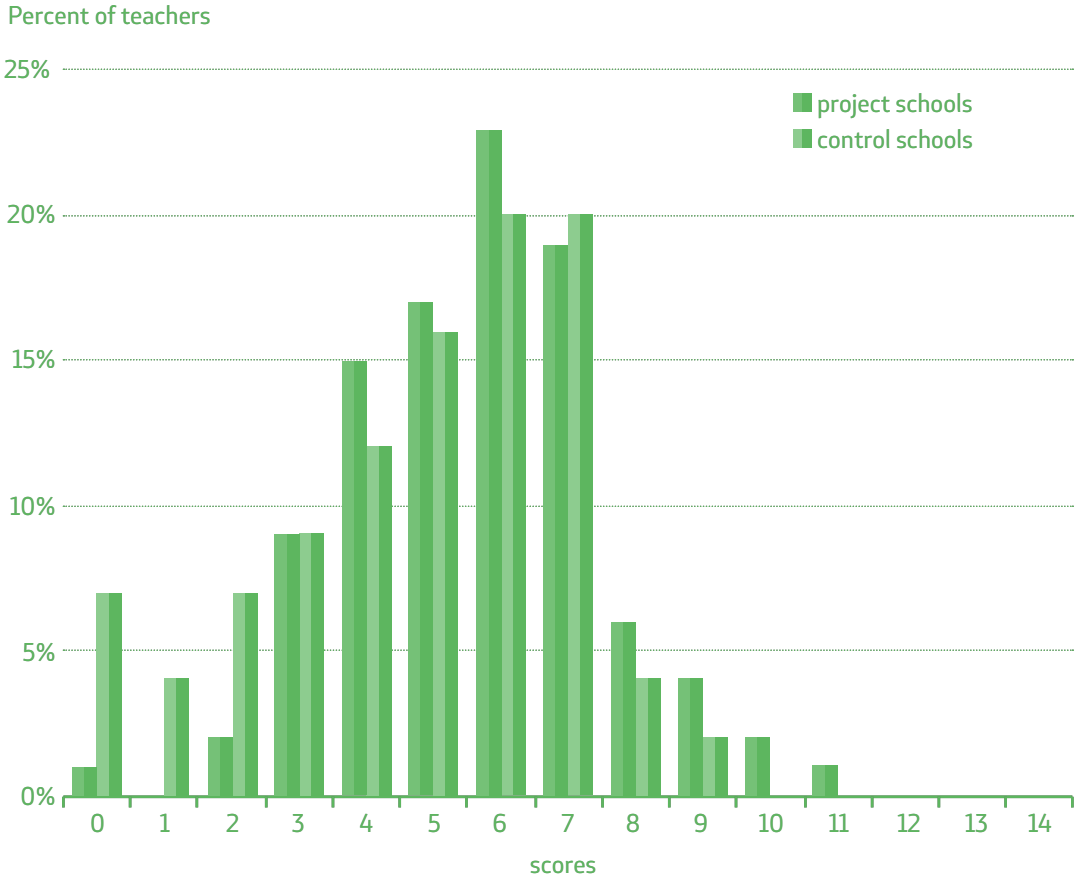
1.2. Pedagogical knowledge of teachers

WAY OF MEASURING

Pedagogical knowledge of teachers was measured by a *Test on the pedagogical knowledge of teachers,* which consisted of 11 teaching situations (out of which one was a cluster one) where the teachers in giving their respnses were expected to use their pedagogical knowledge. All items, except one task, were multiple choice and they required selecting **the true or the best** response.

The maximum possible score that the teachers could achieve was 14.

The distribution of the achieved scores of all teachers (teachers from the project schools and from the control schools) in 2012 is given below.

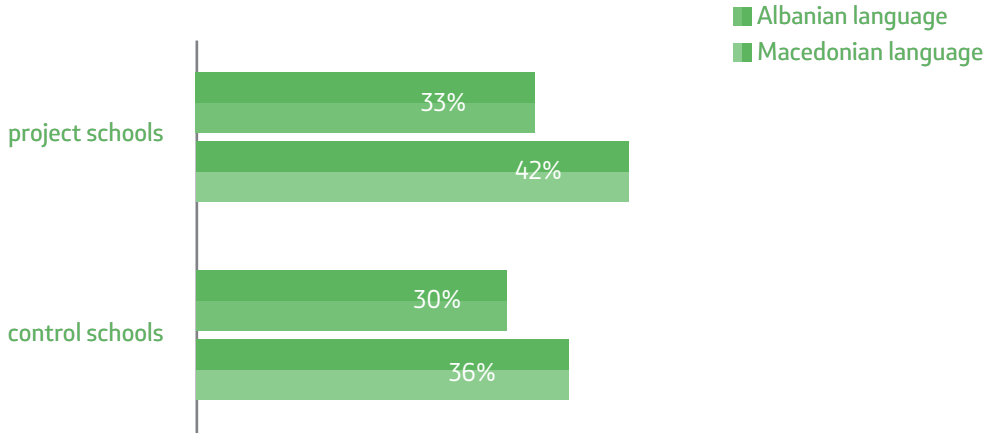


Graph 1. *Scores of the teachers from the project and from the control schools, on the test of pedagogical knowledge in the investigation of 2012*

- The teachers from the project schools achieved higher score on the test of pedagogical knowledge than the teachers from the control schools. In 2009, there were no considerable differences of the achievements on doing the pedagogical test between the teachers from the project schools and those from the control schools.

- The average score at the test achieved by **the teachers from the project schools** is 5,6 points, i.e. the average percent of doing the test is 40%. The largest is number of teachers (32, i.e. 23%) who scored 6 points. Out of 138 tested teachers, six teachers did not respond correctly to none of the requirements, and no one of the teachers responded correctly to more than 11 out of 14 requirements.
- The average score of the 138 tested teachers **from the control schools** is 4,8, i.e. the average percent of doing the test is 34%. The highest achieved score by three teachers is 9 points, 9 teachers did not respond correctly to no one of the requirements, and the largest is the percentage of teachers (20%) who responded correctly to 6 or 7 requirements.

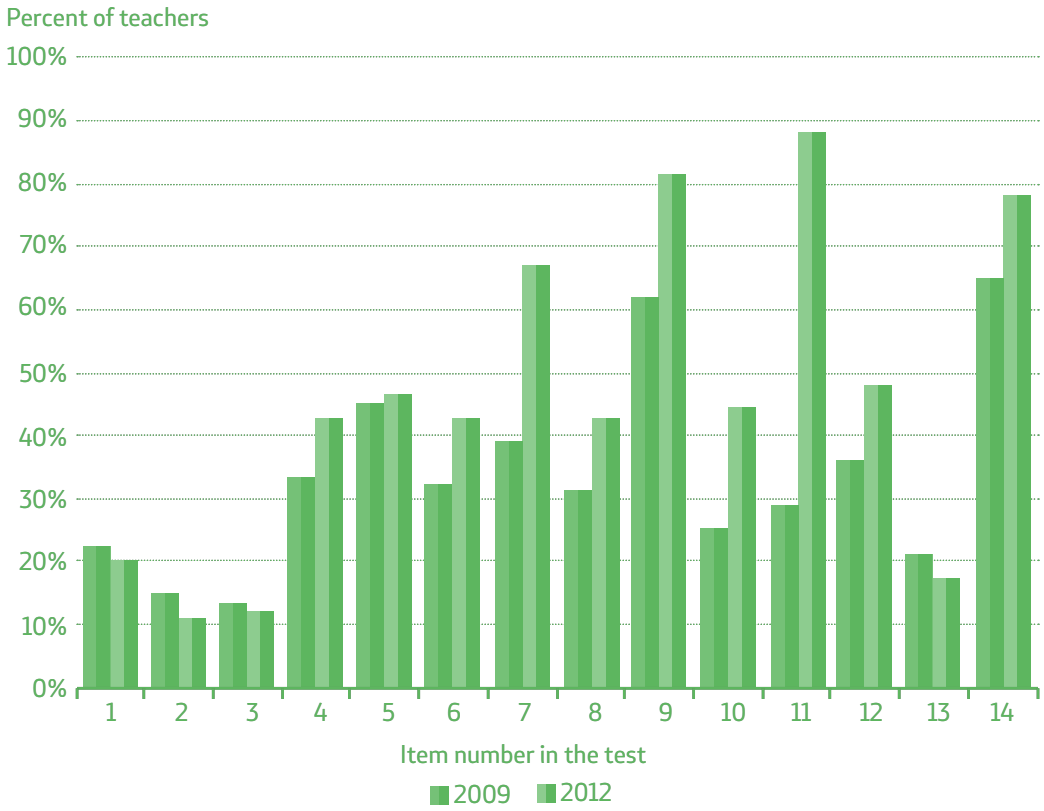
Although preliminary, the language of instruction was not a factor that was considered as a potentially influental upon the pedagogical knowledge of teachers, the more detailed processings have shown that the teachers doing the instruction in Macedonian languuage, both in the project and in control schools, have shown higher results. The average percentage of doing the test according to the languages of instruction is shown on Graph given below.



Graph 2. *The average percentage of doing the test on pedagogical knowledge in the project and in the control schools (according to the language of instruction) in the investigation of 2012*

The differences in successfully doing the test of pedagogical knowledge are bigger with the teachers from the project schools, and together with it, the teachers doing the instruction in Macedonian language were significantly more successful. In 2009, in the project schools there were minimal differences compared to those of the teachers doing the instruction in Albanian language. The achieved indexes can be useful in planning the support to the Project.

The graph given below presents the comparative results on the test of pedagogical knowledge of teachers from the project schools in 2009 and in 2012 on each requirement of the test.



Graph 3. *Teachers' results from the project schools related to the requirements of the test on pedagogical knowledge in 2009 and in 2012*

The Table 2, gives descriptions of the requirements of the items and their relationship with the Ten Principles promoted by the Project *Thinking* Mathematics. The number of the requirement in the table corresponds to the number of the requirement in the Graph 3.

Table 2. *Description of the requirements in the Test on pedagogical knowledge and the relationship with the 10 Principles*

Item number in the test	Require-ment	Description of requirement	Link with the Ten Principles
1	1	Reaction to a response by a confused student when he explains whether number 18 is even or odd. The requirement is to assess whether the reaction of the teacher saying: *Now which is even, and which is odd?* is an adequate one or is not. It is expected to consider the response as not adequate.	Formative assess-ment – feedback.
1	2	Reaction to a response by a puzzled student when he explains whether number 18 is even or odd. The s requirement is to assess whether the reaction of the teacher saying: *If I have understood you well, you think that 18 is an odd number,* is an adequate or is not. It is expected to consider the response as adequate.	Formative assessment – feedback.

1	3	Reaction to a response by a puzzled student when he explains whether number 18 is even or odd. The teacher is required to assess whether the reaction of the teacher saying: *Come on, once again, think well?* is an adequate one or is not. It is expected to consider the response as not adequate.	Formative assessment – feedback.
1	4	Reaction to a response by a puzzled student when he explains whether number 18 is even or odd. The teacher is required to assess whether the reaction of the teacher saying: *Can anybody with own words say what Philip has said?* is an adequate one or is not. It is expected to consider the response as adequate.	Formative assessment – feedback.
2	5	Reaction to a good response given by a weaker student. Out of 5 offered reactions of the teacher, one is to be selected, where the adequate is the one that says that the task is done well and the student is required to explain to other students how he/she has done it.	Assessment of learning and requiring explanation for mathematical thinking.
3	6	Introducing multiplication of a two-digit number. A description is given how two female teachers introduce multplication of two-digit numbers. One of them is showing an open task (which could be done by multiplication) in which, at first, it is expected from the students to do it in any way, and the other one is showing a direct approach in using the multplication. It is expected to select the first approach.	Using various strategies.
4	7	Strategies for teaching the operation multiplication. A description is given how three female teachers introduce multiplication. Teacher A asks the students that they learn first the multiplication table; teacher B, first, explains to them the process of multiplication, teacher C sets up a problem which is solved by multiplication and leaves to the students to look for solution by using manipulative aids. It is expected to select the last strategy.	Using manipulatives.
5	8	An incorrectly done task is given, of addition of two three-digit numbers with „writing down“ where „the writing“ of the numbers that are „memorized “ is not clear. Out of the four possible reactions of the teacher, it is expected to select the one which requires additional explanation.	Accepting and motivating the use of different procedures in solving tasks.
6	9	Help is given to a student who has problems in addition with transition. Out of the three offered ways for help, it is expected to select the one which refers to using manipulatve aids.	Using intuitive knowledge and using manipulatives.

7	10	A solution of a task in two steps (multiplication and division) is given. The mistake is done in the second step. Out of the four offered reactions from the teacher, it is expected, as the best one, to be selected that one which in some other way (drawing on a piece of paper with small squares) would present the solving.	Balansing between conceptual and procedural knowledge, solving textual task.
8	11	A different way in determining a sum of three two-digit numbers. Different ways of solving it are given by three students. Out of the offered four reactions of the teacher, it is expected that he selects that one which shows that he would leave to the students themselves to do it in a way that is easiest for them, or that the student who has mathematically a most efficient solution will ask him to show how he has done the task.	Accepting different strategies in doing the tasks, recuirement of explanation for the solution.
9	12	Change of the content planned for a teaching hour. In a situation when the students are excited by a living situation which has many potentials for learning numbers, it is required, out of the four offered ways of a reaction to the teacher, related to the change of the planned contents, to select that one which says how to make use of the newly arisen situation.	Adjusting the time in introducing the content adequately to the interest of students.
10	13	Out of the 5 offered manipulative aids, it is expected that as the most adequate one for explaining the grouping of tens and units with small children, be selected the one with the plastic boxes for counting.	Selection and use of adequate manipulatives.
11	14	On a drawing with 10x10 points, the teacher should denote which correct answer he is expecting from students when multiplying 4 x 3. As a correct answer, accepted is only that one in which 4 rows with 3 points are denoted.	Using manipulatives, graphical presentation, linking of concrete and symbolic presentation.

- The percentage of teachers from the project schools, who, as the most adequate ones, would select the reaction of the teachers in accordance with *Thinking* Mathematics, has increased in a larger number of teaching situation, and somewhere it has considerably decreased. The average test result in 2012 was 40%, and in 2009 it was 33%. The difference is statistically significant. In the non-project schools there are no differences in the average test result in 2009 and in 2012.
- Largest improvements (over 15%) in the selection of the most adequete response, are found at the requirements which in the above table are numbered as: 7, 9, 10 and 11. Two of these requirements refer to using manipulative aids, which was very emphasized during the trainings, and which the teachers most probaly, through their practice, have considered it as being useful. The third requirement refered to accepting various strategies

in doing the tasks by the students, which, together with the training of the students to use various approaches and ways of doing the task, was, also, in the focus of the trainings. The last one of these requirements refered to the balancing of the conceptual and procedural knowledge, where the best response contains also using a piece of paper with small squares, which in the trainings was often recommended as a mean that helps in inderstanding concepts.

Below, as an illustration, is given a task (on the graph it is the item with number 10), in which a great progress has been achieved, though, still, the larger number of teachers (over 50%) have selected some of the answers that are not in accordance with the *Thinking Mathematics.*

Task

Marko, a student from Grade III did this task:

Ivan wants to share his chocolate bar with Ace and Ana.
The chocolate bar has 6 rows of 4 cubes.
How many cubes would each one of them get?

He did it in the following way:

$6 \cdot 4 = 24$ $24 : 3 = 7$

Which one of the following actions should be the best for the teacher to do? Choose one answer.

A) To check if Marko knows that division is the opposite operation of multiplication .....(34%)

B) To ask Marko to draw it on a piece of paper with small cubes .....(44%)

C) To tell him to check the response .....(15%)

D) Something else .....(0%)
(write what)

In the above task, next to each answer is given the percent of the teachers who have selected it. The correct answer (B) is most oftenly selected and it was selected by 44% of the teachers (in 2009, it was selected by 23% of the teachers). However, still, one third of the teachers have selected the answer (A), which focuses itself on procedural knowledge (accuracy in division is checked through the opposite operation – multiplication), and not through the attempt to check what is the reason for the wrong result, whether the student has understood the concept of multiplication or of division, and enable the student, once more, with the help of graphic presentatiions, to show and check the process of his/her own consideration and to determine his/her own fault (to which refers the answer B). To a similar approach, refers also the selection of the answer C as the best one.

Most poorly is solved the task number 1, that refered to formative assessment – giving feedback information which had four requirements. With the alternatives A and C, the correct answer is „no“ and with the alternatives B and D, the correct answer is „yes“. Particularly small is the number (less than 20%) of correct answers in relation to the adequacy of the reactions of the female teachers 1, 2 and 3.

Task

The teachers with the students from Grade 3 worked on even and odd numbers.

They have agreed that even are those numbers that can be divided by two without remainder.

Philip selected the number 18.

Philip: Well... if we divide it with 2... it is possible with 2... we get 9... and it is an odd one...

I think even...

The teacher can react to his answer in various ways.

Next to each of the offered reactions mark „yes” if you consider it as an adequate one, or „no“ if you consider it as an inadequate.

	Yes	No
A) Teacher 1: Now which one is even, and which one is odd?	 1	 2
B) Teacher 2: If I have understood you well, you think that 18 is an odd number?	 1	 2
C) Teacher 3: Come on, do it again, think well!	 1	 2
D) Teacher 4: Can somebody, with own words, say what has Philip said?	1	 2

A great number of teachers did not give answer to one task, which in the processing was considered as an incorrect answer. This data points out to the uncertainty of the teachers in giving adequate feedback information.

Formative assessment was part of the training, but it was unsufficient both in its duration and its depth. During the in-school support visits, the teachers very often were pointing out that they do not feel sure enough in the formative assessment, and a large number of teachers answered that they need further support in this field (see the findings in the part Acceptance, implementation and further support, in this report).

CONCLUSION

- Pedagogical knowlege of teachers in project schools has increased in relation to the investigation of 2009. This points out to understanding of the principles of *Thinking Mathematics* and to the increased competences for selection of teaching procedures.
- However, the average answer to the requirements of the test (for a large number of items) is low and it can serve as an indicator for that, in which areas the teachers should get professional support.
- The teachers that do the instruction in Albanian language in the project schools have shown a considerably lower level of pedagogical knowledge in accordance with the *Thinking Mathematics*, which means that they need a reinforced support.

1.3. Teachers' expectations from students related to the achievements in Mathematics

WAY OF MEASURING

Teachers' expectations related to Mathematics' knowledge of students were measured with 2 questions:

- The importance they give to the 13 objectives from Mathematics curriculum until the end of Grade 3.
- To what extent they agree with the statement that students in their class could achieve more than that prescribed by curriculum.

Results are presented in arithmetical means or percentage and comparisons are made between the answers of the examinees from the project schools in the investigation of 2012 and in the investigation of 2009.

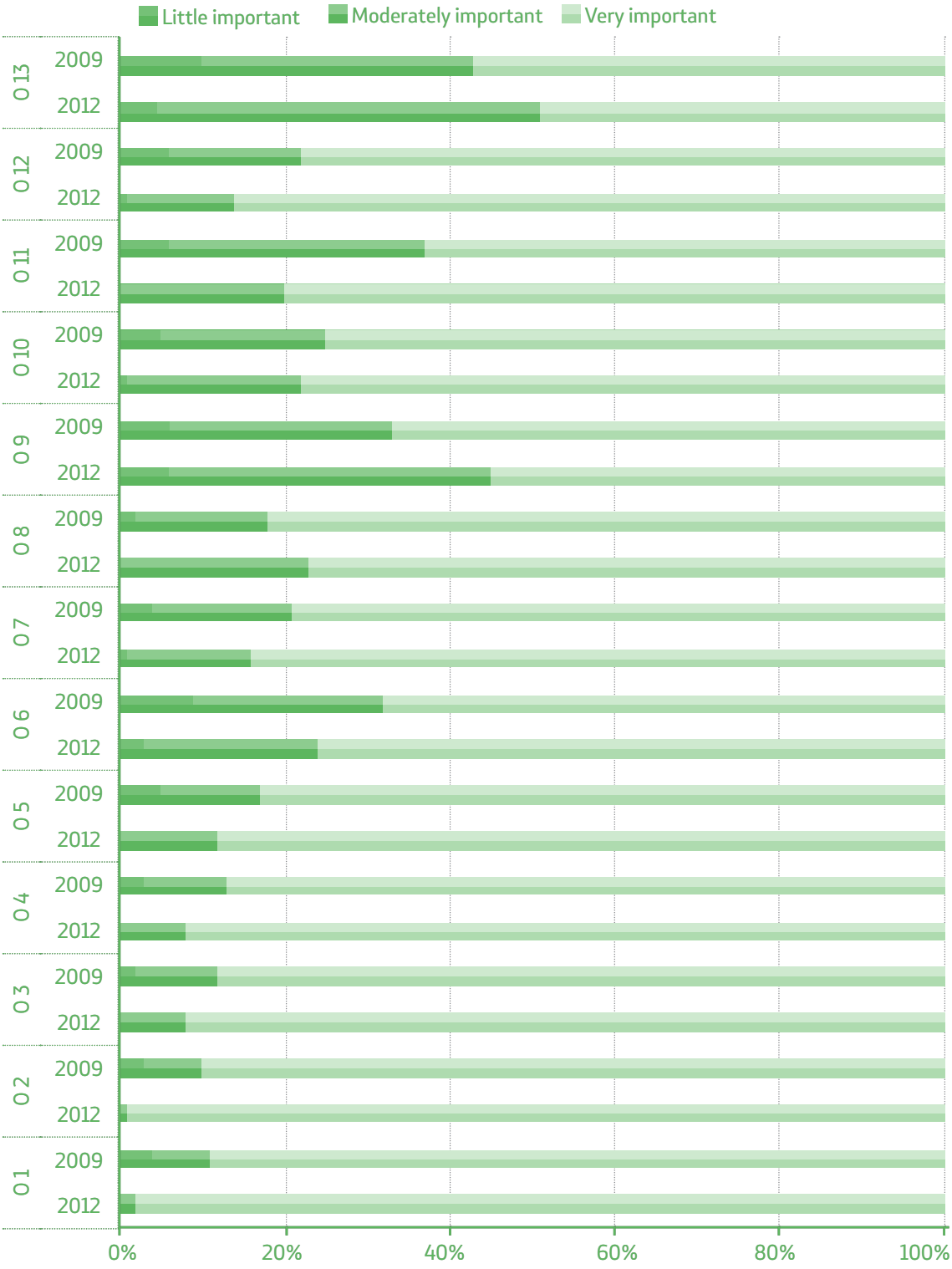
1.3.1. The importance of meeting particular objectives prescribed in the curriculum by the end of Grade 3

The instruction has been carried out, to a great extent, by the implicit curriculum realized by the teacher, i.e. the way how he/she has understood the objectives and he's/she's opinion about their importance. The objectives to which teachers gave responses and objectives link with the Ten Principles are given in the following table.

Table 3. *Relationship of objectives until the end of Grade 3 with the 10 Principles*

	Objective	Link with the Ten Principles
1	To understand addition and its importance.	Balance between conceptual and procedural knowledge.
2	To understand substraction and its importance.	Balance between conceptual and procedural knowledge.
3	To understand multiplication and its importance.	Balance between conceptual and procedural knowledge.
4	To understand division and its importance.	Balance between conceptual and procedural knowledge.
5	To use the properties of operations.	Procedural knowledge.
6	To understand the decade system of numbers.	Number concept.
7	To understand basic properties of natural numbers.	Number concept.
8	To be precise in counting.	Procedural knowledge.
9	To use common procedures in doing numerical tasks.	Procedural knowledge.
10	To notice paterns in Mathematics.	Conceptual knowledge.
11	To be able to solve a problem task using different strategies.	Using various strategies in solving.
12	To use the four basic mathematical operations, the properties of the operations in doing a textual task.	Procedural knowledge
13	To estimate the rezult.	Conceptual knowledge.

The importance (presented in percents) that teachers from project schools give to particular objectives in the instruction of Mathematics in 2012 and the importance they were giving to them in 2009 is shown in the following graph. [4].



Graph 4. *The importance that teachers from project schools were giving to particular objectives prescribed in the curricula*

4 The number of the objective in the graph corresponds to the number in Table 3.

- Almost all of the stated mathematical knowledge/skills were considered as being important by the teachers in the measuring of in 2009 and in the repeated measuring (in 2012).
- In the project schools, it is evident that there is a considerable shift between the two measurings to the importance given to understanding the basic mathematical operations (addition, sustraction, multiplication and division) and to making use of the properties of the operations, where the category - little important – in 2012, is not present at all.
- Also, there is a great increase in the percentage of teachers who give moderate or great importance to the ability of solving tasks no mater of the way by which you come to solution (in 2012, there are no teachers that consider that this aim is a little important).

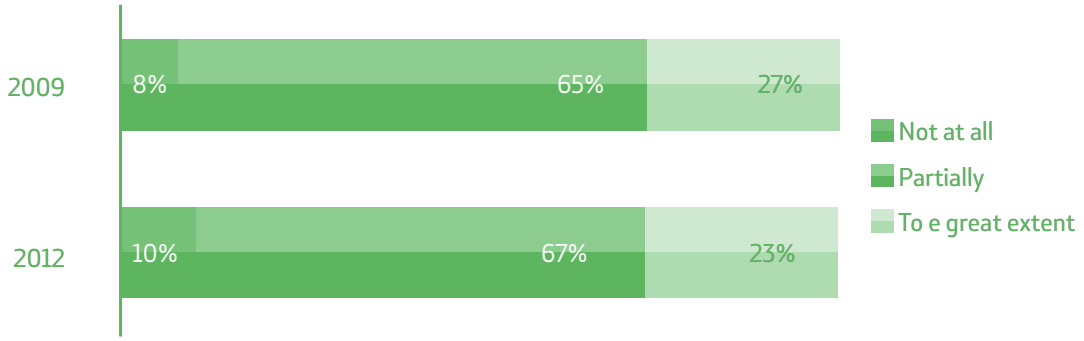
Compared to 2009, the importance that teachers give to foreseeing the result of the estimation and to paterns (schemes, rules, and models) in Mathematics has increased. But, there are still some teachers that consider that this is of little importance. So, although less than that compared to the measuring in 2009, still as more important are considered to be the technical skills and the precision (procedural knowledge).

CONCLUSION

- Compared to 2009, in the measuring done in 2012, there is an increase in the importance that teachers from project schools give to the objectives. This could mean that the expectations of teachers are greater.
- The responses of teachers in this part could point out to which objectives greater attention should be devoted during the training in the schools, where the teachers have not been trained and , as well as during the visits, to give support to teachers in the project schools.

1.3.2. Expectations of teachers about achieving the curricula requirements

One of the findings, from the international experts analysis of Macedonian curricula requirements, was that the expectations concerning the curricula reaquiremens are low (see the information on the Project in part 2 of this Report). On the other hand, in many investigations of the factors for higher achievements, there is a correlation found between the expectations of teachers and the achievements of students. Therefore, we asked the teachers about their expectations of the students in their classes to achieve more than that prescribed by the curriculum. Their answers are shown on the following graph.



Graph 5. *Expectations of teachers from project schools about the achievements of students in their classes*

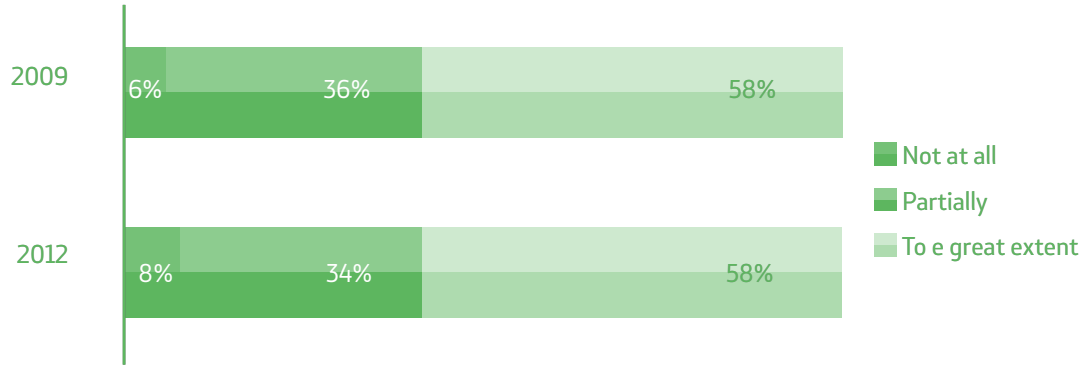
The majority of teachers (67%) agree partially that the students in their classes could achieve more than that prescribed by the curriculum, and one fourth of them agreee to a great extent that students could achieve more. This has changed minimally between the two measurings, most probably because the teachers are based more to that which as expectations is prescribed by the curriculum, than by the individual abilities and skills of the students.

CONCLUSION

- Generally, the majority of teachers do not expect that their students could achieve more than that prescribed by the curriculum.
- There is no difference in the expectations from the students of the teachers from the project schools in 2009 and in 2012.

1.4. Feeling of methodological freedom in the implementation of the curricula

In introducing innovations to the teaching methods, of particular importance is for the teachers to consider that the curricula enable them the needed freedom of methods. The opinions of the examined teachers are presented in the graph given below.



Graph 6. *Opinion of teachers about freedom of methods in curricula*

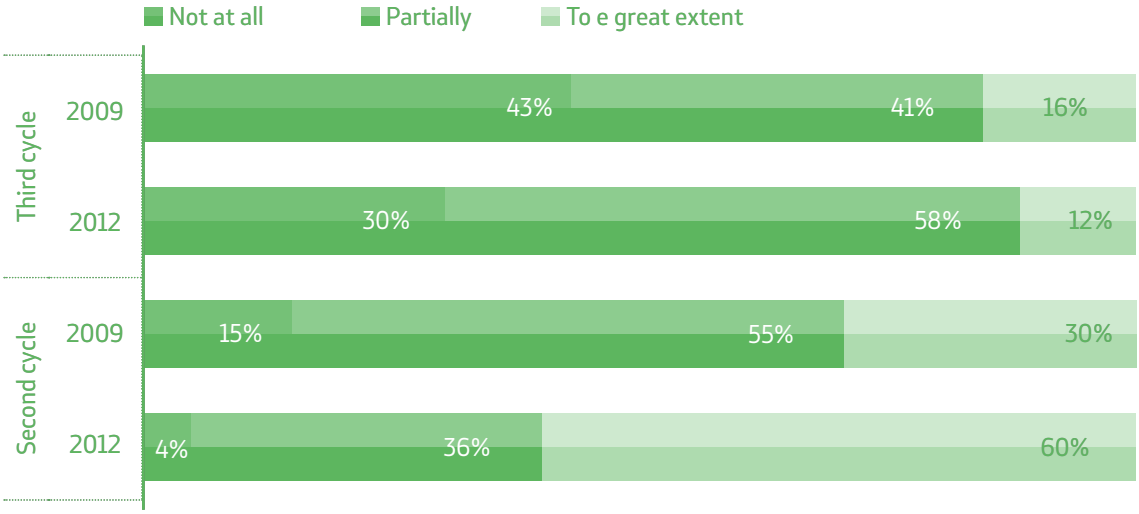
More than half of teachers consider that the curriculum to a great extent enables them to choose the approach in teaching that they consider to be an adequate one. However, about 40% of the teachers consider that the curriculum enbles them to do it partially. There is no difference in the opinion of the teachers in the investigation of 2009 and in the investigation of 2012.

CONCLUSION

- The majority of teachers consider that they have freedom with regard to the using the methods for performing the instruction.
- During the in-school support visits and in the training for teachers who are to be trained, the teachers should be encouraged to use a variety of methods and to undertake responsibility for the achievements of students.

1.5. Familiarity with the Mathematics's curricula in the subsequent cycles

Familiarity with the expected outcomes from learning Mathematics until the end of Primary Education is considered to be important, also, in the implementation of the curricula in the firts cycle. An indicator for the ongoing state in our schools, are the issues on familiarity with curricula for the subsequent cycles. The responses of the teachers from the project schools, who during the training discussed on the importance of the familiarity with the expected outcomes, are shown on the following graph.



Graph 7. *Familiarity with the Mathematics' curricula for the subsequent cycles of the Primary Education*

- Almost all of the teachers from the project schools, partially or to a great extent, are familiar with the curricula for the second cycle (from Grade 4 to Grade 6), and two thirds of the teachers are familiar with the curricula for the third cycle (Grades 7 to 9).
- However, there is still 30% of teachers who are, not at all familiar with the Mathematics' curricula for Grades 7 to 9, and 4% of the examinees said that they are not at all familiar with the curricula for Grades 4 to 6.

The issue that teachers are better informed about the curricula for Grades 4 to 6, is probably due to the fact that almost all the trained teachers in the period between 2009 and 2012 have implemented the curricula for the entire first cycle, and part of them have implemented the curricula, also, for Grade 4 and Grade 5.

The increase of 13% with the familiarity of the curricula for the third cycle, is probaly due to the increased cooperation between the teachers from the grade teaching phase and the Mathematics' teachers from the subject teaching phase.

CONCLUSION

- Many more teachers from the project schools, compared to the measuring in 2009, stated that they are well familiar with the Mathematics' curricula for the subsequent education cycles, and particularly for the second cycle of the Primary Education.

2. TEACHERS' KNOWLEDGE OF MATHEMATICS

Research findings shows that the way in which the teacher implements the teaching and the effects of it (achievements of students), depends of how much the teachers are familiar with the contents they are teaching about (Muijs&Reynolds (2002); Ball&Bass (2000), according to VanderSandt S. (2007)), where there is a direct relationship between the familiarity with the contents and the teaching practice (HorisonResearch (2008)). Also, raising the level of teachers' knowlege of Mathematics can bring to a change in the way of teaching where teachers make a more profound correlation with the methodical and the pedagogical knowlege (Ormrod and Cole (1996), according to VanderSandtS (2007)).

Due to the stated reasons, and because the training of the teachers in the project *Thinking Mathematics* had elements that contained Mathematics' knowledge and understanding of particular contents related to the way of thinking of students, in 2009 and in 2012 a test was used in which were given situations where the teachers had to use their Mathematics' knowledge in order to give a correct response.

The same instrument (test) was used in the two investigations. Below is given a short description and are presented the results of the second investigation, as well as the comparisons between the two investigations.

WAY OF MEASURING

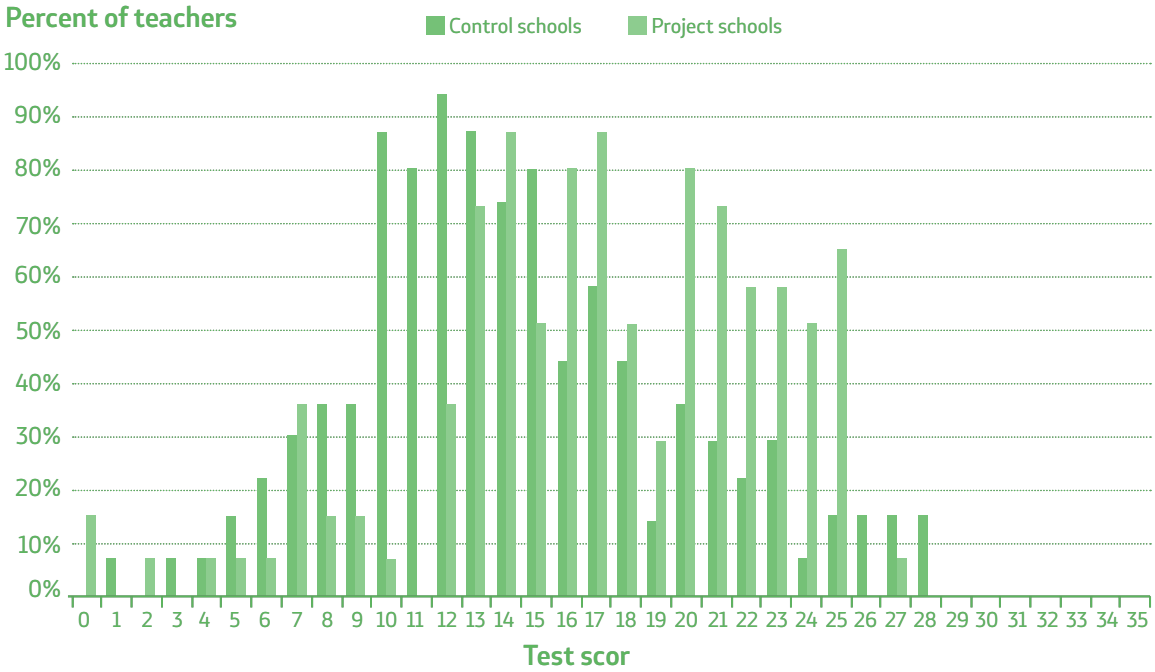
The test on teachers's Mathematics knowledge consisted of 15 tasks, out of which 8 are clusters with 3 or 4 items each. All these tasks were used to measure the knowledge and the skills in the areas of: concept of number (9 requirements), operations and properties of operations (19 requirements) and problem solving (6 requirements).

The test tasks, although designed for the teachers, from the aspect of mathematical content, do not exceed the curricula for Mathematics in Primary Education, i.e. the expected results of the students at the end of Grade 6.

The limitation because of the small number of tasks (items) in the areas of the test, do not allow generalization of the conclusions for the entire mathematics. But still, on the basis of the results we can conclude about the level of this knowledge and the skills of the teachers which were explicitely measured by a corresponding task on the test.

2.1. Achievements of teachers on the test in Mathematics' knowledge

The maximum number of scores on the test was 35. The distribution of the achieved scores on the test in mathematical knowledge of teachers from project schools and from control schools, is given in the table below.

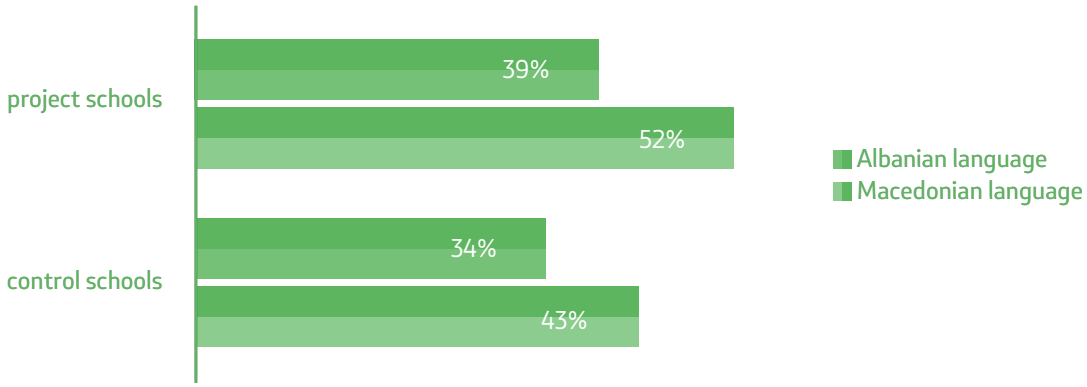


Graph 8. *Scores of the teachers from the project and from control schools on the test in Mathematics' knowledge*

- Teachers from the project schools achieved a 8% higher average score on the test in Mathematics' knowledge, than the teachers from the control schools, which represents a difference that is statistically significant. In 2009, there were no differences in the successfulness in doing the test between the teachers from the project schools and those from the control schools.
- The average score on the test for the **teachers from the project schoos is** 17 points, i.e. the average percentage of doing it well is 49%. The highest achieved score is 27 points (two teachers), and the highest is the percentage (number) of teachers that have 14 and 17 points. Out of 138 tested teachers, two have not responded correctly to neither of the requirements.
- The average score on the test of 138 tested teachers **from the control school** is 14 (out of the maximum of 35), i.e. the average percentage of doing the test well is 41%. The largest is the percentage of teachers who have a score of 12 points, and the highest achieved score is 28 points (achieved by two teachers).

Although, the language of instruction was not considered to be a factor that has influence upon the scores on the test in Mathematics' knowlege of teachers, detailed processing of the data by various subgroups have shown that the teachers

implementing the instruction in Macedonian language in the project schools have higher results. The graph bellow shows the average achievement on the test in 2012 according to languages of instruction. In the project schools, the teachers doing the instruction in Macedonian language have 13% higher results than the teachers doing the instruction in Albanian language. The teachers doing the instruction in Macedonian language in the control schools also, were more successful in doing the test in Mathematics' knowledge, but the difference in the average achievement is smaller (9%).



Graph 9. *The average percentage of solving the test in 2012 in the project and in the control schools according to language of instruction*

- The results on the test in Mathematics' knowledge in the **project schools** have been compared also in the two testings, which have shown that the results of the teachers are 15% higher in 2012, than in 2009 (in 2009 the average result on the test was 34%, and in 2012 it was 49%).

The table below gives a description of all of the requirements on the test in Mathematics' knowledge, i.e. the expected response and the percentage of teachers from the project schools who responded correctly to the requirements in 2012 and in 2009.

Table 4. *Description of the requirements of the tasks on the test and the achieved result in percents*

Area	Description	% in 2012	% in 2009
Numbers	*It is not correct* that 0 is an odd number.	77	73
Numbers	*It is not correct* that 0 is a place holder in writing a number.	34	35
Numbers	*It is correct* that number 8 could be written as 008.	33	15
Numbers	*It is not corrects* that number 72 could be written as a sum of tens and units in 3, 6 or 7 ways, but *it is correct* that it could be written in 8 ways.	51	17

Operations Characteristics	Three procedures of multiplying numbers 35 and 25 are given, and for *each one* it was to be determined that *is adequate* for multiplication of any of two natural numbers.	A-45, B-61, C-45	A-10, B-43, C-45
Numbers	In a given teaching situation, it was to be discovered that *the student does not understand* the meaning of the place value.	52	49
Operations Characteristics	In adding two negative numbers, *it is correct that you* never get a positive number.	58	27
Operations Characteristics	In subtracting two negative numbers, *it is correst that you never get* a positive number.	21	19
Operations Characteristics	In multiplying two negative numbers, *it is correct that you always* get a positive number.	76	49
Operations Characteristics	In dividing two negative numbers, *it is correct that you always get* a positive number.	41	12
Numbers	Selection one correct out of five offered options of *graphic presentation of* multiplying the number 2 with number 0, 3.	37	33
Operations Characteristics	An incorrect solution is given of a task with subtracting two two-digit numbers. Out of the offered responses *you should select the reason* according to which the student came up to such a solution.	70	61
Operations Characteristics	6 decriptions of adding numbers 43 and 48 are given, with the names of *the steps that should be related to the corresponding description*.	32	13
Проблеми	*In three different (quite correct)* ways of solving a textual problem task, you should *determine their correctness.*	A-83, B-79, C-81	A-55, B-47, C-69
Operations Characteristics	Four explanations from students who show understanding and application of the distributive property are given. You should determine *the corectness (incorrectness) of each of the explanations*.	A-74, B-35, C-32, D-30	A-43, B-20, C-29, D-17

Operations Characteristics	A procedure of a student is given, in which he multiplies two two-digit numbers in four steps. You should determine which one of the properties: commutativity, associativity and distributivity, *is used or not used* in the given solving.	A-60, B-81, C-22	A-33, B-59, C-20
Проблеми	With three examples, explicitely is given the rule that you should *discover and write it down algebraicly*.	29 on the first task, and 12 on the second	7 on the first task, and 6 on the second
Operations Characteristics	Two examples are given, of "unusual" subtraction and explanation by a student about the way how he did them. The explanation of the student is correct, but it is said in a "childish way", insufficient for a mathematical language. You should detect that an ordinary borrowing *has been used and explained*.	58	63
Operations Characteristics	Estimations of the addition of 4 three-digit numbers are given, you should *select* that one which leads closest to the correct result.	71	52
Проблеми	In a given problem situation, the beginning steps in doing the task are given, you should *determine* and give response how the task could be solved correctly. Then you should *give explanation of the response*.	A-59, B-47	A-41, B-27

The results show that the greatest achievement of teachers is noted in:

- determining the number of ways in which a given double digit number can be written down as an addition of tents and units (improvement of 34%);
- estimating whether a given specific procedure in multiplying with two digit numbers is adequate to any of two natural numbers (improvement of 35%); and
- determining that with the addition it is never correct that „two negative numbers make a positive one" (improvement of 31%), and that with multiplication this statement it is always correct (improvement of 33%).

In the requirements of the task, where a "stepwise" multiplication of two double digit numbers is given, in comparison to 2009, 27% more teachers determined that in doing the task a commutative property is used, and 22% that an associative property was used. But, only more than 2% of teachers determined that a distributive property is not used.

There is also a very small progress in the results on the task where the teachers were expected, to select out of 5 possible answers the adequate one for graphic presentation in multiplying a whole number with a decimal number (only 4% have selected the correct presentation).

Although, compred to 2009, there are 22% better results on the first requirement, and 6% better on the second, again most poorly is solved the task (given below) with the requirement for a symbolic i.e. algebraic presentation of the rule (it was to be recorded (a-1)+a+(a+1)=3a or something else adequate).

Task

Check whether:

$4 + 5 + 6 = 3 \cdot 5$

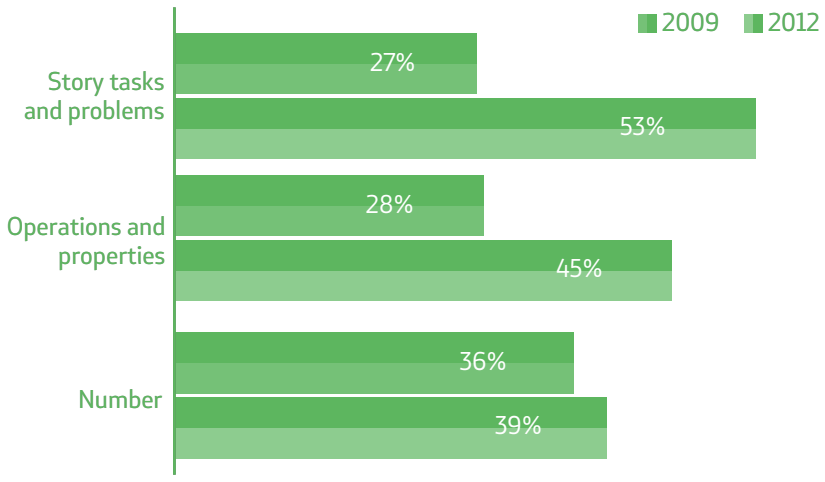
$39 + 40 + 41 = 3 \cdot 40$

$125 + 126 + 127 = 3 \cdot 126$

Write down, with words, your explanation of the rule.

Write down, with words, your symbolic (algebraic) description

In the additional analysis, we grouped the results of the teachers according to the areas from which we derived the tasks in the test. The graph below presents the average solvability in the project schools in 2009 and in 2012 according to mathematical areas.



Graph 10. *Average result on the test in the project and in the control schools according to areas in Mathematics in 2012 and in 2009*

- In all of the areas, the results in 2012 are higher compared to those in 2009. Largest progress (21%), between the two measurings, is noted in the average results on textual tasks and problem situations, and the smallest is the progress (6%) in the area of numbers.

In both investigations the requirements in the tests by which the pedagogical knowledge and the mathematical knowledge and skills of teachers were measured, were explicitly connected with the principles, the techniques and the contents of the Program *Thinking Mathematics in the early grades*. Although the training of teachers, to a great extent, refered to the pedagogical knowledge, the use of various approaches and techiques in teaching, and even offering particular models that can be directly applied in the teaching of Mathematics, it was noted that the teachers in the project schools have shown smaller progress – 7% on the test in pedagogical knowledge compared with 15% on the test in mathematical knowledge. The primary objective of this Program is not to strengthen the mathematical knowledge of teachers (it is supposed that they posses it), but to insist on that how the teachers ought to work with the students, and at the same time to strengthen the understanding of particular concepts. Most probably, the work of the teachers with their students in order to explain and explicate the concepts, the notions and the rules, has provided improvement in their own mahematical knowledge.

This gives directions that during further giving support to the trained teachers, their attention should be directed more to the pedagogical aspects of the Program.

CONCLUSION

- The mathematical knowlege of teachers in the project schools has increased in comparison to that of the investigation in 2009. This denotes better uderstanding of the principles and the techiques of *Thinking Mathematic* so that their use in the work with the students enabled a different way of thinking and solving teaching situations in which the mathematical knowlege is needed.
- There are rquirements in the test which still have low average of solving, and this can serve as a sign for the areas in which the professional support should be strengthened (for ex. in Numbers).
- The teachers that carry out the instruction in Albanian language in the project schools have shown a considerably lower level of mathematical knowledge, which means that they need more intensive support.

3. SUPPORT TO THE TEACHING OF MATHEMATICS AT SCHOOL LEVEL

The support to changes that are introduced in teaching and learning Mathematics by the use of the principles and techniques promoted by *Thinking* Mathematics is a very important factor for the successfulness of the implementation of the Project and in enabling higher achievements of students.

WAY OF MEASURING

The level and the way of providing support was examined only in the peoject schools through interviews with school principals and school pedagogues/ psychologists. They were both given two groups of questions. The first 6 questions refered to the teaching of Mathematics at the school level, and were given also in the investigation prior the beginning of *Thinking* Mathematics. This time, in addition to the description of the state, an opinion was asked about whether something has changed in the last three years. Those questions were related to:

- satisfaction with the achievements in Mathematics in the grade teaching;
- expectations from students' achievements;
- other activities for promoting the teaching of Mathematics (in addition to those of the Project);
- cooperation among teachers concerning the teaching of Mathematics, and
- equipment for the grade teaching of Mathematics.

The other 4 questions were related to:

- implementation of *Thinking Mathematics*;
- effects from *Thinking* Mathematics and
- own involvement of the schol principals/pedagogue/psychologists in the implementation of *Thinking Mathematics*.

The responses from the interviews are analyzed qualitatively.

In the questionnaire for teachers, also, there were 2 questions related to the cooperation and the support in introducing innovations in general and the support to *Thinking Mathematics*. The responses to these questions were processed quantitatively and are shown graphically.

3.1. Considerations about the achievements of students in Mathematics

In the two investigations (in 2009 and in 2012), in the interview with the school principals and pedagogues/psychologists, the questions were asked related the expectations and the satisfaction with the students' achievements in Mathematics. We considered that the level of expectation is a motivator for changes and a possible indicator of the changes that has happened.

3.1.1. Satisfaction with achievements of students in Mathematics

Two thirds of the interviewed (11 pedagogues/psychologists and 12 school principals out of a total of 31 interviewed) expressed satisfaction with the achivements in Mathematics of the students in the grade teaching. In the investigation in 2009, the number of school principals and pedagogues/psychologists that were satisfied with the achievements of students was smaller. Then half of the interviewed were satisfied with the achievements.

An indicator for the success, according to the opinions of the school principals and of the pedagogues/psychologists, as it was the case also in the previous investigation, are the achievements of students on contests, and more often, then in the previous investigation, they consider that the good results are due to the engagement of the teachers and to the contemporary methods of teaching. Six out of the interviwwed explicitly stated the *Thinking* Mathematics as a reason for achieving better results.

> *I am very pleased, because I consider that new ways of learning Mathematics are applied through plays, with new forms and methods and open classes.* (school principal)

> *I am very pleased with the results. I see the success in winning the awards at "Kengurce" and in other contests. ... the grade teachers work in acccordance with the methods and teachiques required by the project "Thinking Mathematics". Also, the teachers successfully give meaning to tasks, for the students with special educational needs. The students present that what they have done, and we are present at their presentations.* (psychologist)

Partial satisfaction was expressed by five of the interviewed. For the majority of them, the achievements are partially good because all the teachers have not been trained for the new approaches in the teaching of Mathematics or because "they do not want to accept the new obligations and are not sufficiently engaged ".

No satisfaction was expressed by three examinees. Dissatisfaction is due to various reasons: the composition of students, over abundant curricula, irresponsible approach of teachers.

Those interviewed who responded that they are partially satisfied, or are not satisfied with the achievements, are the school principals and pedagogues/ psychologists of the schools with the instruction in Albanian language, and in one school in which there is a considerable number of Roma students.

It is interesting to note that the responses of the school principals and of the pedagogues/psychologists from the same school are more balanced in comparison to the state prior to the implementation of the Project, when the school principals more often, than the pedagogues/psychologists, were satsfied with the achievements of students.

3.1.2. Considerations about the possibiliy of improving the achievements

The high expectation, according to a great number of investigations, are one of the unfluental factors for higher achievements.. With regard to this, as it was the case in the previous investigation, *all* the interviewed consider that the achievements of students *could be improved*. The majority of them consider believe that *the key* for improving the achievements is in the hands of *the teachers.* They should use contemporary teaching methods. Five of the interviewed mentioned tha use of the principles and techniques *from Thinking Mathematics.*

> *Yes, surely, (the achievements could be improved). It depends mostly on the teachers, how much they want and how much attention they devote to the teaching everyday, because it is the teacher who leads the students. I ascertained it with my students 2-3 years ago, when the Project Thinking Mathematics was introduced in Grade I. The students had an opportunity to acquire many strategies for logical thinking and with it to gain solid mathematical knowledge. (*pedagogue*)*

Some of the interviewed pointed out that in order to achieve higher achievements, crucial for achieving higher achievements, are the trainings of the teaching staff.

> *Yes, I think that the students in our school are able to achieve higher success in Mathematics both in the grade teaching and in the subject teaching. The reason for this my eastimate, is that teachers are to a sufficient level trained and ready, through adequate seminars and workshops, attended by them, to work at a significant level and provide help to the students in acquiring the knowledge. (school principal)*

In contrast to the previous investigation, the technical facilities and the material means, as a factor for greater achivements of students, are practically not mentioned. Two of the examinees mentioned the regular maintenance of the *complementary and of the additional instruction.*

As in the previous investigation, only a small part of the examinees consider that the key factor for higher achievements is the aptitude of students for Mathematics, as well as the support by parents.

CONCLUSION

- The majority of the managing staff of the schools, most oftenly school principals, is satisfied with the achievements in Mathematics in the grade teaching cycle. The satisfaction is not often based on sound indicators of success, as are those by objective internal measurings, comparisons with national standards or comparisons at international level.
- The conclusions about the effects from *Thinking Mathematics* were mostly made indirectly, through the statements of the teachers or through the changes in the behaviour of students towards learning Mathematics.
- Though, in the previous investigation, it was recommended to the schools to develop a different culture in measuring their own success, which should be promoted also within the framework of the Project, it was implemented, hence there are no changes in the focus of the schools of improved activities towards the achieved effects.
- The increased number of school principals and pedagogues/psychologists that are satisfied with the achievements is in corrolation with the improved results of the students in relation to the previous measuring of their achievements.
- On the other side, the considerations that the achievements could be improved primarily by improved work of the teachers, and often by more consistent implementation of the approaches *fromThinking Mathematics* is a good ground for further sustaining and suuport that are expected by the managing staff.

3.2. Considerations about the implementation of Thinking Mathematics

More than half of the school principals and pedagogues/psychlogists (16 out of 31) are satisfied with the implementation of the activities from *Thinking Mathematics* in their schools. They base their satisfaction on the fact that the implementation was according to what was planned for, that the instruction is more interesting for the students or that they have got such information from the teachers that are carrying it out.

From the talks with the teachers, students and their parents, it can be noted that there is a satisfaction with the implementation of the activities from the Project, and the results will be noted in a longterm application. (school principal)

We all wanted to be included in the Project „Thinking Mathematics ". Our teachers have had a great interest for the Project, made efforts to work efficiently, asked for continuing proffesional support by our trainer, and she was was giving generous support. Truly, the work is done according to the Project. (psychologist)

Those that are partially satisfied with the implementation of the Project, as a reason for it, point oit that the Project is not implemented by all teachers, because this is something new to them and that they have difficulties in the implementation.

Three of those interviewed are not satisfied, and one school principle responded that he doesn't have information because he is not monitoring the implimentation, but that „it is done by the pedagogues/psychologists ".

3.2.1.Difficulties that they were facing in the implementation

Almost all of the schools were facing particular difficulties in the implementation of the Project (only three of the interviewed school principals and pedagogues/ psychologists responded that they had no difficulties). Mostly the difficulties were related to the lack of material means and teaching aids (5) or to the resistence of the teachers to the changes (5).

> *There were many difficulties – how to find the time that would suit all the teachers to share experiences, to discuss about the concepts. Impact had to be put on some teachers to accept the Project. There is a consideration that the Project was imposed very spontaneously and for a long time certain teachers were giving resistance to the implementation.* (pedagogue)

They managed to overcome the majority of the difficulties that were of organizational and proffesional nature.

> *At the beginning, there were not enough materials to work with, and that was surpassed by producing aids and materials done by the school caretaker, and by providing ready made materials.*(psychologist)

Part of the difficulties were more of objective nature (for ex. the fixed arrangement of the compjuters in the classroom, which did not allow adequate organization of the space) that the school could not cope up with. For certain schools, the lack of time or work in combined classes represented a particulr difficulty.

3.2.2. Findings about the effects of the Project Thinking Mathematics upon the knowledge of students

The considerations about the effects of *Thinking Mathematics* were explicitly asked from the interviewed school principals and from the pedagogues/psychologists. It is interesting to mention that only one third (11) of the interviewed have expressed directly about the effects upon the knowledge of students. It is probably due to the practice in introducing changes not to monitor sistematically the effects from them. In the first phases of the implementation, the focus is directed to the activities. Therefore, the school principals and the pedagogues/psychologists had only partial and indirect considerations about the effects of the *Thinking* Mathematics which they got from the teachers or on the basis of the visits to their classes.

A consderable improvement has been noticed, in relation to the prevous one, but a lot of time is needed to achieve bigger success. (pedagogue)

The estimate is that the effects of the Program are positive from the aspect of acquiring knowledge by students, where of particular importance is the acquiring of skills for logical thinking. (school principal)

When they talked about the effects, the school principals and pedagogues/psychologists very often quoted the changes in the behaviour of students to learning Mathematics. They expressed opinions that the students are more active (5 examinees), more motivated (3 examinees), more clever (1 examinee), and that Mathematics is not a "bugbear" for them (1 examinee).

The effects upon the knowlege of the students are positive, students learn with biger interest, through plays, they can do tasks in many ways and create mathematical problems by themselves. (pedagogue)

There is an effect upon students, they are more active, they want changes, and the classes are more interesting for them. . (pedagogue)

Two of the interviewed responded that they have no evidence because they do not monitor the implementation, and one school principal responded that *Thinking Mathematics* has not been implemented.

CONCLUSION

- The majority of school principals and pedagogues/psychologists are satisfied with the implementation of *Thinking* Mathematics because it is implemented as it was planned and has induced positive changes with the students. The others are partially satisfied primarily because it has not been implemented by all of the teachers or they did not succeed to overcome the difficulties which they were facing.
- Almost all schools were facing difficulties in the implementation, that most oftenly refered to the lack of teaching aids and materials, and rarely to a lack of motivation on part of the teachers. There are schools that stated that they by themselves have overcome successfully the difficulties that they were facing.
- Generally, almost all of the examinees spoke about the positive changes with the students, either they refer to those that refer only to the motivation, the interest and to the increased involvement of the students, or if they refer to the increased achievemnts in Mathematics. Such considerations are based almost exclusively on non-systematic observations, and not on evidences from measuring and making comparissons of the achievements of students.

3.3. Support to the instruction of Mathematics

The support to the instruction of Mathematics in this investigation is expressed through the cooperation of the grade teachers among themselves and the cooperation with the subject teachers of Mathematics, as well as through the engagement of the pedagogues/psychologistsand of the school principals in the implementation of the project activities.

3.3.1. Cooperation of teachers related to the teaching of Mathematics

Mutual cooperation between the teachers of the grade teaching cycle

Two thirds (22) of the interviewed school principals and pedagogues/psychologists consider that the teachers in the grade teaching cooperate between themselves quite a lot, mostly through staff meetings, in planning and in implementing open classes, exchange of experiences, with regard to contests and in making didactical materials for the instruction, and five of them responded that in the last three years, the coperation has increased, and four mentioned the *Thinking* Mathematics as a content around which the cooperation takes place

They plan together and implement teaching units in the Project „Thinking Mathematics " in the lower grades and intergrate them with other subjects … They exchange didactical materials, they create together tests, etc. (school principal)

Under the influence of the Project, the grade teachers cooperate more between themselves, most oftenly for the implementation of open classes and the exchange of teaching materials and ideas in the implementation of particular contents.(psychologist)

Near one third (7) of the interviewed consider that the teachers cooperation is insufficient.

The teachers in the grade teaching cooperate, but such cooperation is not sufficient, and in most cases it is superficial. The teachers need still more time for an open and sincere cooperation with an aim to achieve higher results at the grade level and at the school level. Many activities in the school are directed to have exchange of experiences, attendance at class hours, sharing of ideas, etc.. (school principal)

Two school principals consider that there is almost no cooperation.

There is no shifting. Each teacher works for himself. Nothing has been changed. (school principal)

In comparison to the consideration in the investigation of 2009, almost *all* of those interviewed responded that the teachers from the *grade teaching ccoperate among themselves considerably*, in the repeated investigation the school principals and pedagogues/psychologists more critically assess the cooperation. Most probably it doesn't refer to a declined cooperation, but to increased expectations in relation to the intensity and to the quality of the coperation , what is usually expected when introducing some change in which all the grade teachers are included.

Cooperation between the grade teachers and the subject teachers

With regard to the cooperation of the grade teachers with the subject teachers, there prevails the opinion (expresed by 18 of the interviewed) that there is a cooperation and it is good. The cooperation is mostly related to various issues of the teaching (for ex. on doing textual tasks, on the passing of the students from grade teaching to subject teaching, and on taking part at the competions in Mathematics.

> *Grade teachers and subject teachers cooperate on: giving help in doing particular mathematical tasks and problems, exchange of information, passing of the students from grade teaching to subject teaching, teaching aids and facilities. (school principal)*

One part of the school principals and pedagogues/psychologists (9) consider that they have certain cooperation, but are not content with its quality.

> *The subject teachers of Mathematics are maximally available for cooperation with the grade teachers, and especially after the nine year education has been introduced Part of the teachers due to shame and negligence move away from the cooperation on textual tasks, on equations and on logical tasks. (school principal)*

> *They cooperate, but I think that the meetings could be more intensive and more qualitative. Sometimes the subject teachers ask for ideas from grade teachers who are very creative. (school principal)*

In relation to the previous investigation, the number of school principals and pedagogues/psychologists that consider that there is no cooperation, has decreased. This time only three of the interviewed expressed such opinion (in 2009, eight of the interviewed expressed such opinion).

> *Subject teachers do not cooperate with grade teachers although the grade teachers want and ask for their opinion.(school principal)*

According to the responses of the school principals and pedagogues/psychologists, it can be concluded that the cooperation of the grade teacher with the subject teachers has improved in the last three years.

3.3.2 Activities for improving the instruction of Mathematics, that have been undertaken by the project schools

To the question: „Have you, beside the activities in the Project *Thinking Mathematics,* in the last few years, you, as a school, undertaken something to promote the instruction of Mathematics and to improve the achievement in Mathematics in the grade teaching phase? What?" two thirds of the interviwed (22 out of a total of 31) stated that they had some activities. In most cases, it refers to organizing workshops, open clasess, mutual work within the profesional acting bodies and similar activities for *professional further improvement of teachers* and in organizing *competitions and internal testing of students* (as stated by 8 of the interviewed). Five of the inteviewed mentioned the activies for the *Improuving Mathematics and the Natural Sciences* within the USAID Primary Education Project which, also, refered to contemporary approaches in teaching, four of the interviewed stated that they have provided *new teaching tools*, or that they use ICT (two of the interviwed, and three of the interviewed stated they have strengthened and have improved the additional and the complementary instruction.

Yes, the school constantly organizes meetings with teachers and organizes workshops by the team for making teaching aids, exchange of experiences, reviewing and giving support to teachers on various issues and dilemas, supervised classes. (pedagogue)

Yes, the school conducts other activities, also, by organizing competitions and internal testings. Also, it provides teaching aids and other necessary materials for teachers. (school principal)

In relation to the investigation in 2009, the majority of the interviewed stated activities related to the exchange of experiences and to the cooperation of the teachers, and rarely were mentioned the activities that refered to improving the conditions from the aspect of *providing new technical* equipment or teaching aids.

Nine of the interviewed responded that they have not undertaken other activities for the improvement of the instruction of Mathematics, except those from the *Thinking Mathematics.* They explain the issue of not undertaking usually due to lack of financial means.

3.3.3. Involvement of school principals and pedagogues/psychologists in the implementation of the project activities

All of the pedagogues/psychlogists and the school principals from the project schools, except one school principal, are personally involved in the implementation of the Program. Their personal engagement, most oftenly, consists of giving consultative support to teachers (9 of the interviewed), class visits and consultative help (6), taking part in the work of the proffesional teacher's acting bodies (4), providing materials for work (5), support to the dissemination (5), monitoring the

changes in the use of the principles and techniques in the teaching and taking part in the certification (4). A considerable number of school principals and pedagogues/psychologsts (6) were previously grade teachers and trainers in the Project, and truly it was easier for them to be involved in many various activities for its implementation.

I have been involved in the Program Thinking Mathematics as a trainer, and now as a school principal. My contribution consists of professional-consultative help to teachers in the implementation of the Program namely: providing help in planning and in the preparation, in the direct monitoring of classes, in monitoring of the portfolios from the aspect of the implementation of the Program, having constant cooperation with the Bureau for Development of Education . (school principal)

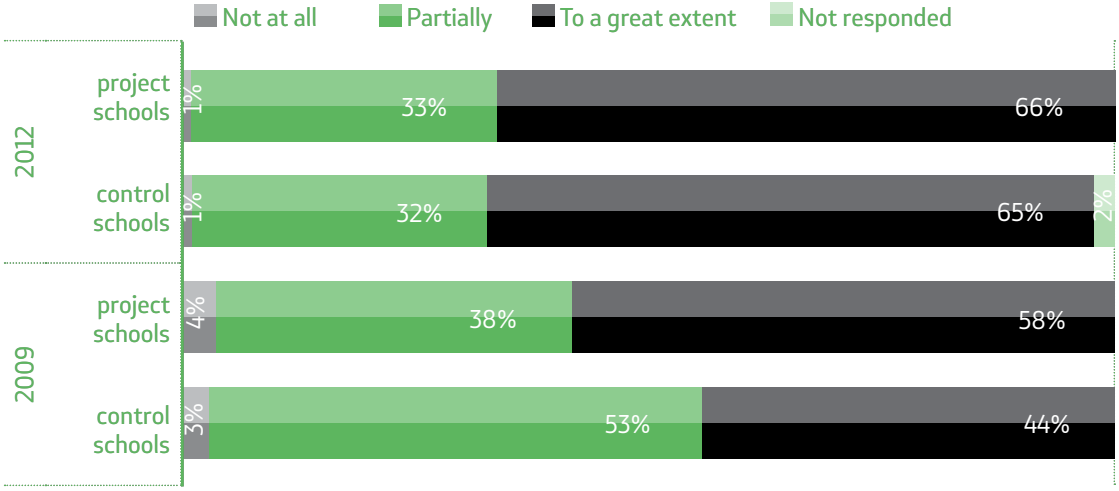
I am a member of the team for giving support to the Program „Thinking Mathematics ", I am giving support to the teachers in planning and in the implementation of the Program ,giving support to the teachers for certification and with my active participation I take part in the work of the professional teachers' acting bodies. (pedagogue)

As a schhol principal, I have been involved in this Program since the beginning of the implementation, as I provided conditions for training of the teachers,I was giving support to the implementation of the curricula, have monitored the implementation, have visited classes, and as a teacher of Mathematics I was giving also professional help to the teachers in their preparation and in the implementation in their classes.(school principal)

The responses of the school principals and of the pedagogues/psychologists point out to their considerable and various inclusion in the implementation of *Thinking Mathematics*. This represents *a big change* in relation to the period before the beginning of the Project when half of the interviewed school principals and pedagogues/psychologists stated that they have no sufficient information about the Project. The teachers from their school applied to become trainers wihout having consultation with the school management, and the schools did not know that they would be included in the first phase, and what would be happening with the Project. Then, a particularly big change has occured in the role which the school priincipals, in meantime, have accepted, because prior to the Project, the school principals were considering themselves only as a logistic support and were planning to accept the managerial role in the Project.

3.3.4. Sense of support to teachers

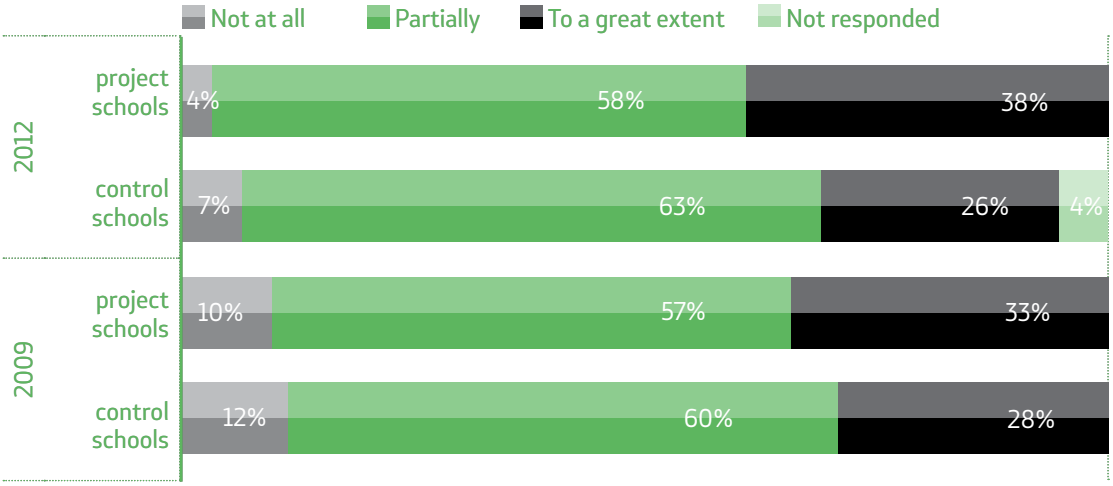
In addition to the responses of the school principals and pedagogues/ psychologists, the grade teachers also responded to a question in the questionnaire, to what extent they cooperate with the other teachers with regard to the teaching of Mathematics. Their responses are presented in the graph below.



Graph 11. *To what extent teachers from project and from control schools cooperate with other teachers*

Two thirds of the teachers, both from the project and from the control schools, responded that to a great extent they cooperate with other teachers in relation to the teaching of Mathematics. In comparisson to 2009, the cooperation in the project schools in 2012, has increased for 11%.

The teachers were asked whether they can count on support in the school for introducing innovations in the teaching.



Graph 12. *Sense of support for changes*

The teachers from the project schools, to a significantly larger number, responded that they can count on the support for introducing changes in the teaching. In 2009, they did not differ among themselves. Such a change is probaly due to the support that the majority of them have received during the implementation of the Project. However, there are still more than 50% of the teachers, who responded that they can only partially count on the support in introducing changes in the teaching.

CONCLUSION

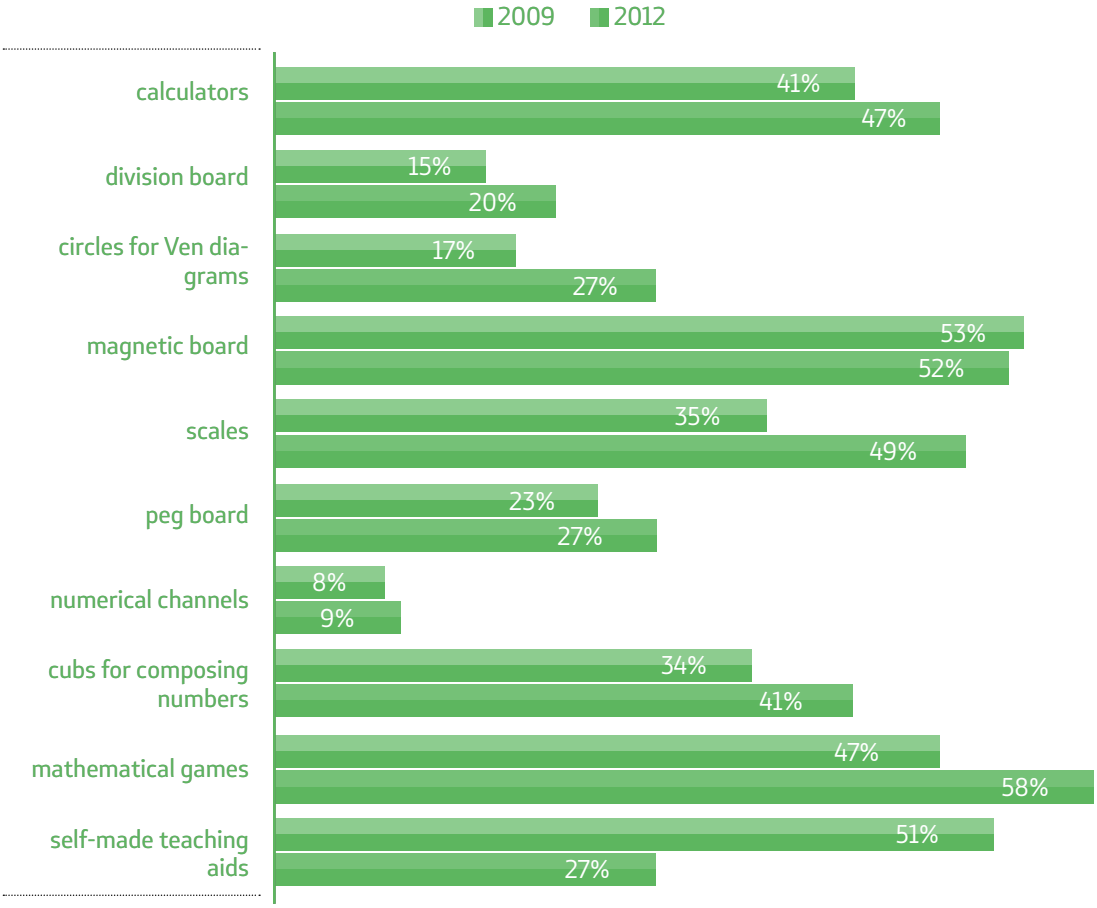
- The cooperation between the grade teachers and the subject teachers, acccording to the responses of the school principals and of the pedagogues/psychologists, as well as to the responses of the teachers, during the implementation of the Project, has increased.
- In the schools, beside the activities that are part of the Project, other activities for improving the instruction of Mathematics, have been conducted, that are mainly in the direction of enlarging and enriching the activities from *Thinking Mathematics.*
- Almost all of the school principals and pedagogues/psychologists, stated a series of adequate ways in which they have been included in the support for the implementation. Although the feeling of the teachers that they can count on the support in the school for introducing changes in the teaching, in the project schools, has improved considerably, nevertheless the majority of them responded that they partially count on such support. This denotes that still it is necessary to work on increased involvement of all in introducing methods' changes in the instruction.

3.4. Equipment of schools for the instruction of Mathematics

One of the Ten Principles in *Thinking Mathematics* is the use of manipulative aids. Part of them could be handy, made by teachers and students, but for some content purposely produced teaching aids are necessary. It is essential to provide abundance of various teaching aids.

From the interviews with the school management staff, it was obvious that part of the pedagogues/psychologists and school directors are not informed about the teaching aids that are used in the instruction of Mathematics in the grade teaching phase. In the next investigation there was almost no interviewed teacher who spoke that the teaching aids are not suitable for the grade teaching phase, because the schools have provided them or the teachers have made them by themselves. In the meantime all students were provided with small lap-top computers.

According to the responses of the teachers about the teaching aids for the instruction of Mathematics that they have in their classrooms, it can be concluded that that the majority of the classrooms are still poorly equiped. In comparison to the beginning of the Project, now the humber of students who have calculation aids, circles with Ven's diagram division board, scales and mathematical plays, has been increased. According to Graph 13, one gets an impression that the number of teaching aids made by the teachers themselves has decreased, but that is probably due to the fact that part of the previously mentioned particular aids had been made by the teachers. This has been mentioned also by the school principals and by the pedagogues/psychologists.



Graph 13. *Equipment of Schools with teaching aids*

CONCLUSION

- The equipment of classrooms has been moderately improved, but still it is not satisfactory. It is not sufficient to support the Thinking Mathematics needs.

4. ACHIEVEMENTS OF STUDENTS

In the investigation at the end of the first cycle of *Thinking Mathematics in the early grades* with an aim to determine the influence of the Program upon the quality of the instruction and upon the achievements of students, we measured also the achievements of students at the end of Grade 3. Within the framework of this indicator, also, the expectations were that the change in teachers in doing the instruction, resulted by the methods and pedagogical approaches promoted by *Thinking Mathematics* in the project schools, would induce a positive change in the results of the students.

WAY OF MEASURING

We based the estimation of the achievements of students upon the achieved scores in doing the tasks that measure conceptual and procedural knowledge, understanding and application of natural numbers, the four basic operations and their properties, as well as in solving textual tasks and problems.

The test for students consisted of 19 tasks (21 items) which measure knowledge and skills in the areas of:

- Concept on number – 5 items;
- Operatons and properties of opeations – 11 items, and
- Problem situations – 5 items.

The limitation from the aspect of the small number of tasks (items) in the mentioned areas of the test, does not allow the generalzations in this Report to refer to the knowledge and skills that were explicitly measured by the test items.

The same test was used in the measuring in 2009 and in 2012.

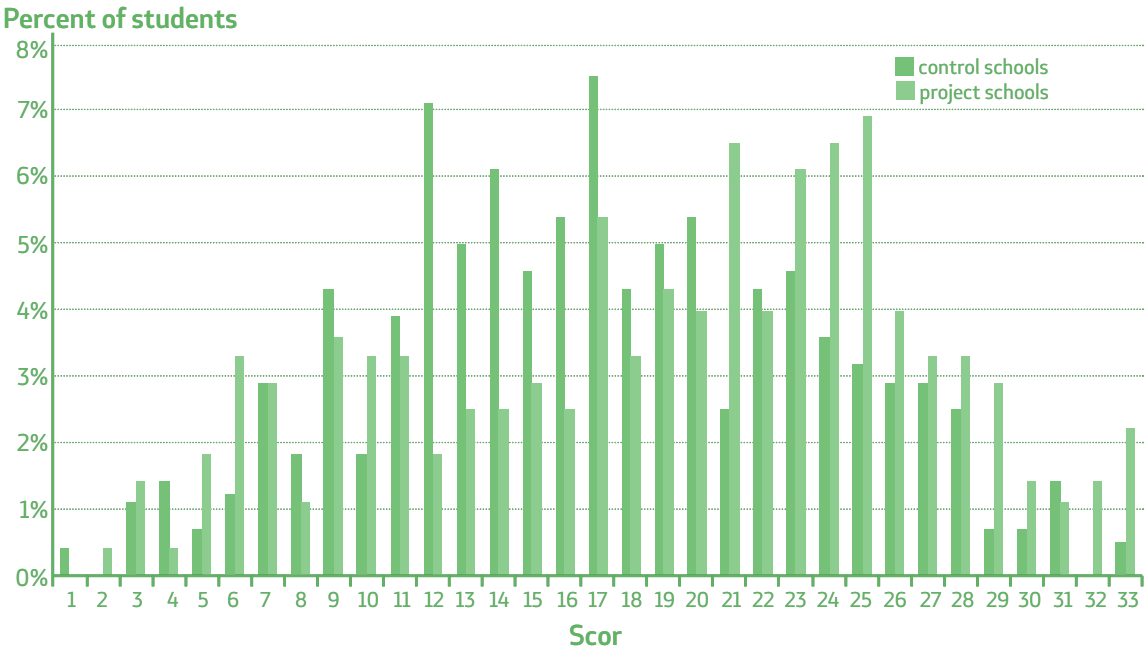
4.1. Achievements of students on the test in Mathematics

The results in this part are presented through the percentage of doing the test well on the whole and according to particular areas, and there, where it is appropriate the achieved score is given. The results in 2012 show that:

- The average score of he **students from project schools** (277 students) is 58%. In these schools the score of the students in the instruction of *Macedonian language* is 60%, and that of the students in the instruction of *Albanian language* is 52%. Between these two subgroups, there is a statistically significant difference in the achievements, though both groups of students were taught by teachers that have been trained in the *Thinking Mathematics,* by the main trainers and in the same period of time, which means that they had the same time to use the acquired knowledge in their work with the students and have received support with the same frequency.
- The average score of the **students from the control schools** (280 students) is 51%, and that of the students in the instruction of *Macedonian languuage* is 52%, and the average score of the students in the instruction of *Albanian language* is 50%. There is no statisticaly significant difference between the students in the instruction of different languages.

The difference of the average score between the the teachers from the project schools and those from the control schools is significant at the level of 0.05.

The graph below presents the distribution of the scores according to the achieved scores on the test.



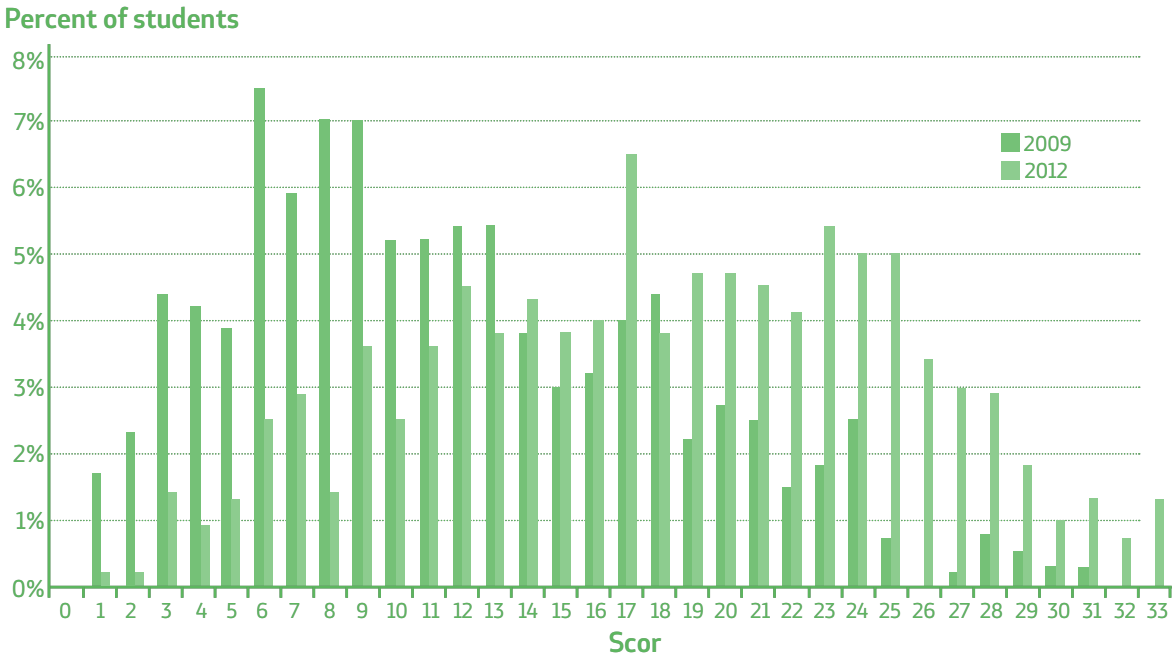
Graph 14. *Scores of the students from project schools and from control schools in 2012*

From the graph it can be noticed that:

- in the project schools, the largest is the percentage of the students who scored 21 points, 23 points, 24 points and 25 points, while in the control schools most of the students achieved 12 points, 14 points, and 17 points;
- the maximum 33 points, was achieved by 2.2% of students from the project schools, and 0.5% by the students from the control schools;
- the distribuition of the rezults in the project schools is bimodal, which means that it separates a group of students with lower achievements from another group with higher achievements, while the results of the students from the control schools are normally distributed, but most of the students are in the part with the average number of points (17).

The main objective of this testing was to see the effects of the Project in the school after its three years of application. Therefore, in this part, comparatively are given the results of the sudents in 2009 and the results in 2012. The general statement is that there is improvement in the achievements of the students between the first and the second measuring.

On the Graph 15 are presented the results of the students in 2012 and in 2009, according to the percentages of students who achieved a particular result (achived number of points on the test).



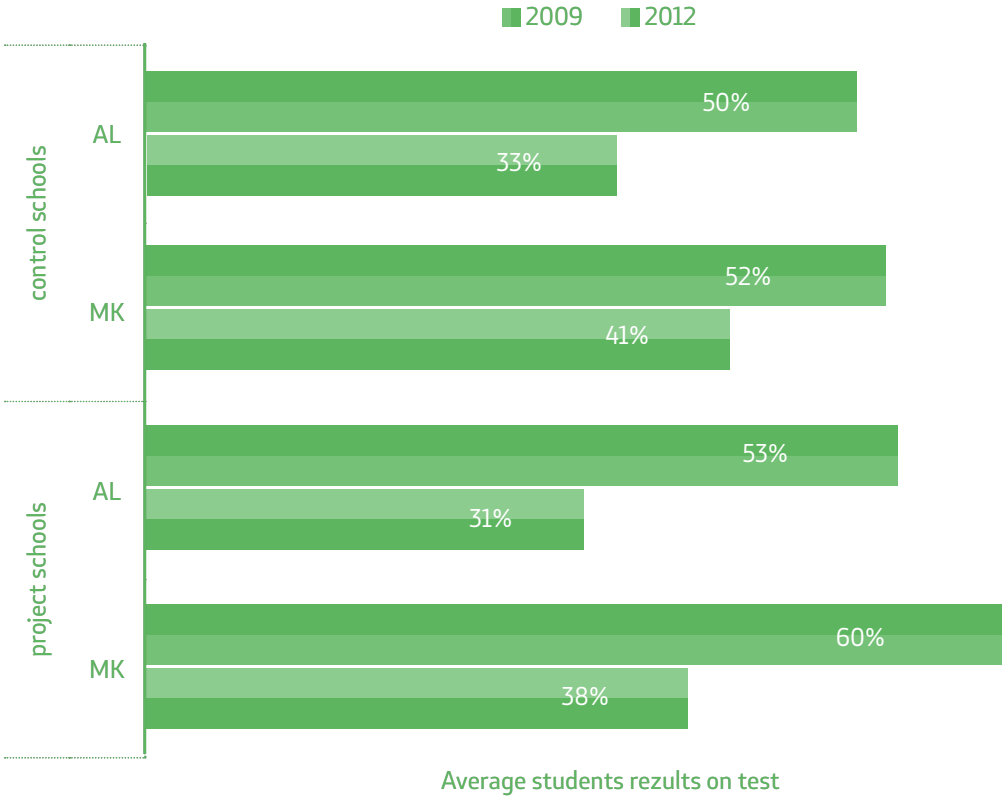
Graph 15. *Scores of students on the test in Mathematics in 2012 and in 2009*

It can be noted that:

- the distribution of the results of the students in 2009 was shifted to the left, and in 2012 it is normal and is little inclined to the right side, which means that more students have achieved a larger number of points;
- in 2009 the largest percentage of students had a result of 6 points, and in 2012 the largest percentage of students had 17 points;
- in 2009 a small number of students had more than 24 points, and a large number had less than 5 points, while in 2012 not a single student has 0 points, and there students that have a maximum number of points.

The achievements of students in the two measurings differ also in the different subgroups, where in 2012 there is a statistically significant difference in the achievements, and in 2009 there was no such a difference.

- The average percentage of solving the test in the **project schools** in 2012 is 58%, and in the measuring of 2009 it was 36%. The achievements of students from in the project schools in 2012 is 22 average percentage higher than those in 2009. The improvement, compared to the results in 2009, is equal both of the students in the instruction of *Macedonian language* and of the students in the instruction of *Albanian language.*
- The average percentage of solving the test in **the control schools** in 2012 is 51%, and in the measuring in 2009 it was 39%. Although the results of the students in the control schools in 2012 are 12% higher than those in 2009, compared to the results in the project schools, the improvement is for 10 average percetage smaller. The improvement in the control schools, is most probably due to the use of contemporary methods' approaches in the teaching of Mathematics, as a result also of the activities with the teachers from Grade 1 to Grade 3 in The Primary Education Project financed by USAID, as stated by the teachers and the school management of the schools.
- According to languages of instruction, the difference between the two measurings in the schools with Macedonian laguage of instruction is 11%, and in the schools with Albanian language of instruction it is 8%.



Graph 16. *Results according to languages of instruction of the students in the project and in the control schools in 2012 and in 2009*

CONCLUSION

- The students from the project schools in 2012 have a 22% higher average results compared to those of the students from the same schools in 2009. This improvement is equal both with the students with the instruction in Macedonian language and with the students with the instruction in Albanian language.
- In 2012, there is an improvement of the results on the test in the control schools also, but it is considerably smaller than that in the project schools.

4.2. Achievements of students according to areas

4.2.1. The results of students on the tasks in the area on Numbers

The average results of the students in the area of Numbers from the project schools are considerably higher, compared to the results from the students in the control schools.

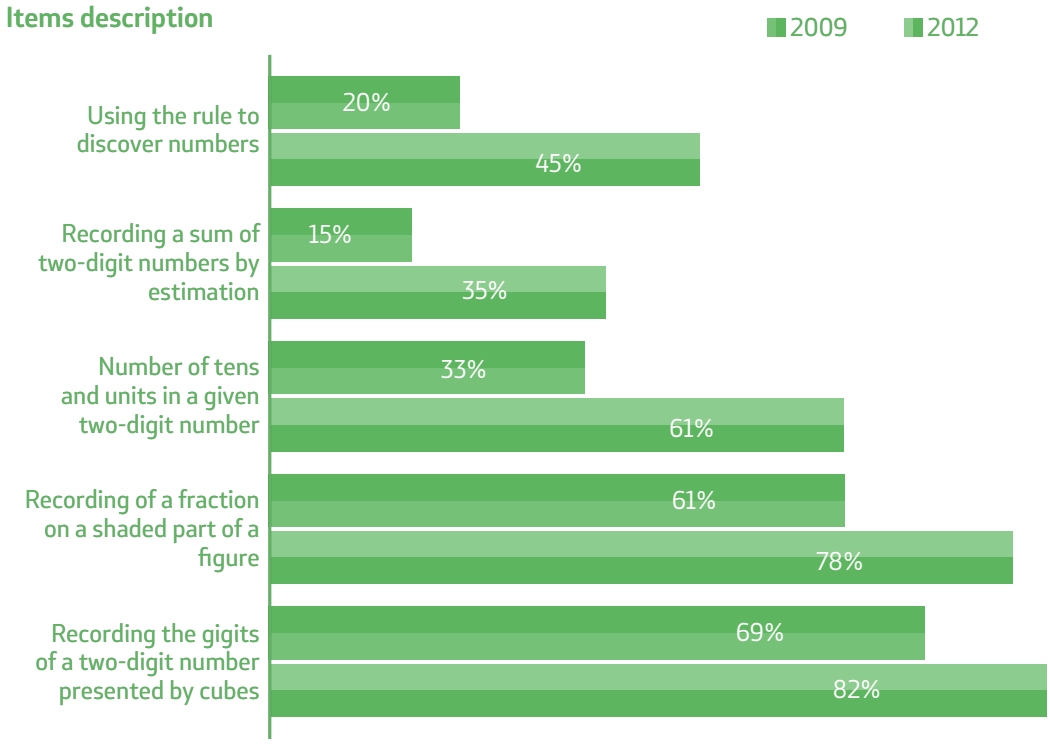
Graph 17. *The results of the students from the project schools and from the control schools on the tasks from the area on Numbers in 2012*

- The average result in the area on Numbers of the **students from the project schools** is 56% correcty solved tasks. The maximum possible 7 points were achieved by 16% of the students, and the largest is the percentage of students (19%) who have 2 points. Out of a total of 277 tested students, 2% have not solved none of the tasks in the area on Numbers.
- The average result of the **students from the control schools** is 48%. The maximum possible number of points was achieved by 7% of the students. In the control schools also, the largest is the percent of students (20%) who have a result of 2 points. Out of 280 tested students in the control schools, 3% have not solved any of the tasks.

The same tasks from this area were solved also by the students in 2009, at the beginning of the implementation of *Thinking Mathematics,* but then there were no differences between the project and the control schools.

- In 2012, in the project schools, the average result of solving the tests by the students is 56%, and in 2009, it was 34%, which is a considerable improvement of the achievements.

The graph below presents the compared results according to tasks between the two measurings with the students from the project schools.



Graph 18. *The results of the students from the project schools in 2012 and in 2009 in Numbers*

- On two tasks, in which in 2009 the largest percent of students had a correct response, the progress in 2012 is greater for 13%, i.e. for 17%.
- On the tasks, in which in 2009 there were much poorer results (solved between 15% and 33% from the students), in 2012 they have better results (for 20% up to 28%).

This improvement is most probably due to the changed way of implementation of the instruction when introducing numbers and to the better understanding of the notion of numbers. In *Thinking Mathematics*, among others, it is insisted on using manipulative aids in composing, representing and decomposing of numbers, as well as of accepting different ways of recording the numbers.

As an illustration for this area, a task is given below, in which it is required from students, out of some given recordings of sums of two-digit numbers, via estimation, to select that recordring which is closest to the correct response. This task in 2009 was correctly solved by 15% students, and in 2012 it was solved by 35%.

Task

Children were adding 29 + 15 + 30 + 26:

Ana: 30+10+30+20

Bojan: 30+10+30+20 and about 20 UNITS

Vlatko: 20+10+30+20 and about 10 UNITS

Goran: 3+1+3+2 TENS and about 1 TEN more

Who of the children is THE CLOSEST to the correct response?

Although in this task, there is a considerable improvement, it can be stated that the students have still difficulties in estimating, as well as in using the rules for discovering numbers. In the curricula for Mathematics in the grade teaching, these contents are not given explicitly, and the teachers probably, during their implementation in the instruction, do not pay sufficient attention to them.

CONCLUSION

- In the area on Numbers, the students from the project schols have achieved higher results. The average percentage of solving the test in the project schools is 56%, and in the control schools it is 48%.
- The results of the students in the project schools in 2012, are considerably higher than the results of the same schools in 2009.
- There are still contents (estimation, models, sequences) in the area on Numbers, to which the teachers should devote greater attention, which would later enable the students to acquire easier the knowledge and the skills from other mathematical areas.

4.2.2. The results of the students on the tasks from the area on Operations and the characteristics of operations

In the area on *Operations* the average results of the students from the project schools are for 5% higher than the results of the students from the control schools. This difference is statistically significant.

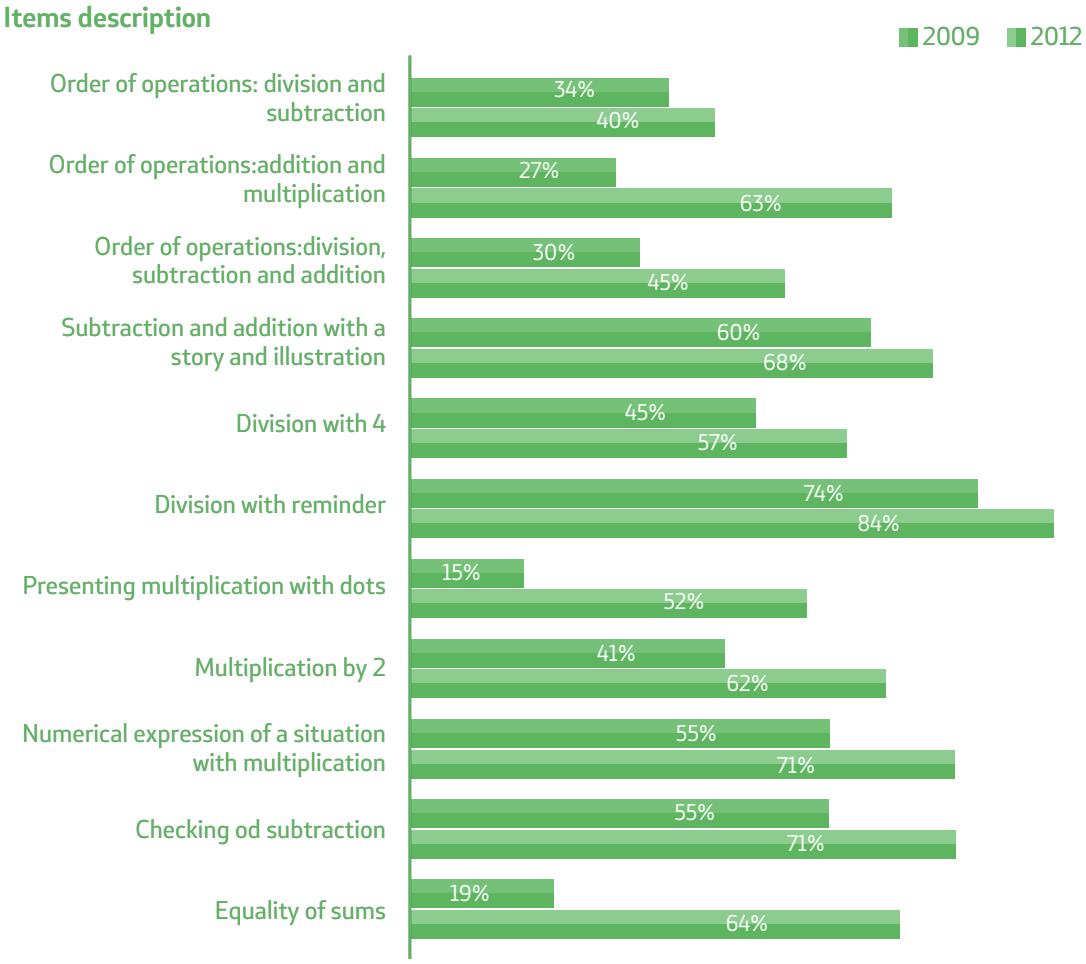
Graph 19. *Results of the students from the project and from the control schools in the area on Operations and the characteritics of operations, in 2012*

- The average percentage of solving the tasks in the area on *Operations and the characteristics of operations* of the students in the **project schools** is 60%. The maximum possible number of points (18) was achieved by 4% of the students, and the largest is percent of students (13%) who have 14 points. Out of the total number of students, 2% did not solve correctly any of the tasks.
- **The students in the control schools** have an average result of 55%. The maximum number of points in these schools is achieved by 2% of the students, and the largest is the percentage of students (10%) who have 7 points. Out of the total number of tested students two students did not solve correctly any of the tasks.

The achievements of students in the area on *Operations and characteristics of operations* in the measuring in 2012, are better than the achievements measured in 2009.

- In the project schools, the average of the solved tasks in this area, in the repeated measuring, is 26% higher than the average of the solved tasks in the same schools, prior to the training of the teachers (in 2012 – 60%, and in 2009 – 34%).
- In 2012, all the tasks from this area were solved more successfully.

The comparable results from the two measurings of the students from the project schools, according to tasks, are given in the graph below.



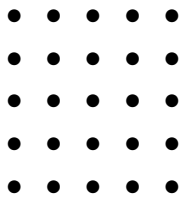
Graph 20. *Results of the students from the project schools in 2012 and in 2009 in Operations and properties of the operations*

- It can be noticed that in this area, largest improvement in the results of the students is recorded in determining the sums which are equal, presenting multiplication with dots and in the order of operations: addition and multiplication.
- Smallest, but statistically significant, is the improvement between the two measurings in: the task which is given with a story and an illustration, used to measure the knowledge of addition and subtraction ; in the task where there is an implicit division with reminder (in 2012 there are 10% more students who solved the task correctly) and in the task where the use of the knowledge about the order of operation in solving a numerical expression with division and subtraction, is checked.

The illustration of the task, which was most poorly solved in 2009, and which in 2012 has considerable improvement, is given below.

Task

Present the multiplication of 4•3 on the drawing below.



The table below gives the possible solutions and the percentage of students who gave correct, partially correct or incorrect response.

	Solution	% in 2009	% in 2012
Correct	ON THE DRAWING, the dots are encircled or somehow denoted in 4 rows with 3 dots OR in 4 columns with 3 dots	10	41
Partially correct	- Aside are drawn and are encircled, or not encircled 4 groups with 3 dots, 3 groups with 4 dots or 12 dots which are grouped in some way - On the drawing are encircled or denoted 12 dots, but without visible grouping where it could be noticed 4 · 3	10	22
Incorrect	- Aside are drawn groups where there 12 dots, but the grouping is not noticable or there is another way of grouping (fo ex. 6 groups with 2, 2 groups with 6) - Any response which is not correct	80	38

The difference between the two measurings with this task indicates that:

- such a way of visual presentation of the multiplication (with a dotted paper or table with sticks) is a new tecnique and approach in the project schools that they began to use it in teaching and in learning the operation Multiplication;
- after the training in the *Thinking Mathematics*, the teachers insist more that their students practice understanding of multiplication;
- in 2009, even 38% of the students, did not even try to solve this task, and the percentage of such students in 2012 is e 8%.

In a specific contest, this task was given to the teachers also. In 2009 – two third of the teachers did not do it correctly, and among them there were teachers that recorded only 4 · 3 = 12. In 2012, almost half of the teachers (49%) did it correctly.

CONCLUSION

- In the area on *Operations and characteristics of operations*, the students in the project schools were considerably more successful. The average percentige in 2012, of doing the tasks in this area in the project schools is 55%.
- There is a statistically considerable improvement in the achievements of students from the project schools in 2012, compared to that one in 2009.
- Beside the considerably higer results for 26 percentage points than that from the previous measuring, and taking into consideration the time also that is devoted to the instruction from Grade 1 to Grade 3, to *Operations and the characteristics of operations*, the results are lower than the expected results prescribed by the curriculum at the end of the first cycle of education.

4.2.3. Results of the students on Textual tasks and problems

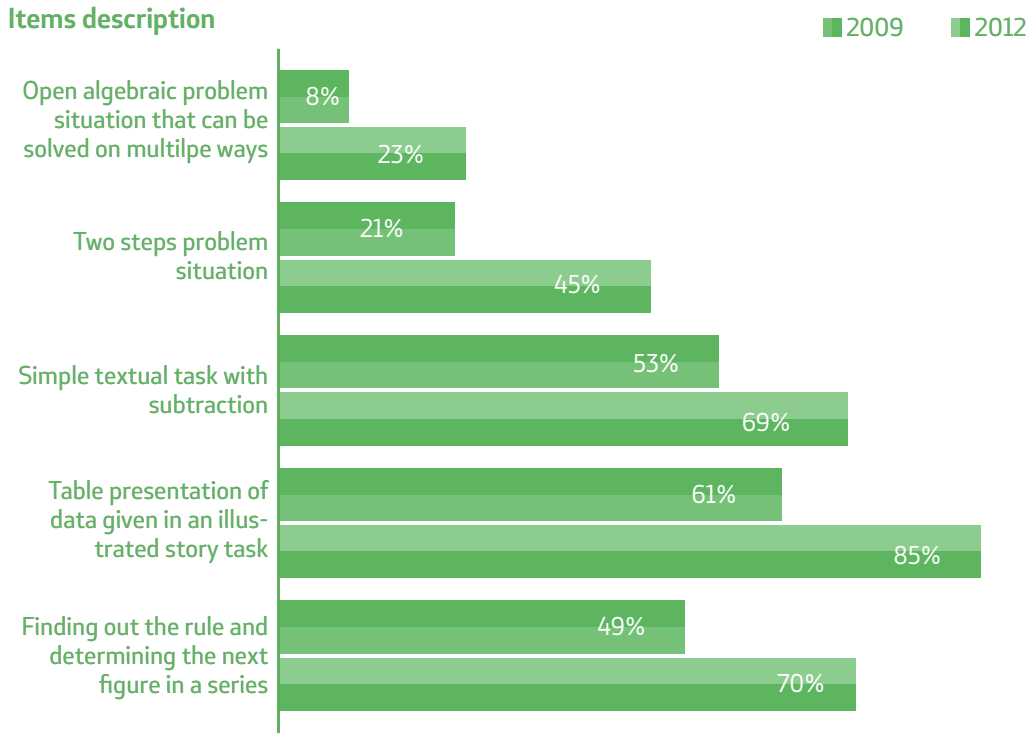
The achievements of the students from the project schools are for 9% better than the achievements of the students from the control schools.

Graph 21. *Rresults of the students from the project schools and from the control schools, in2012*

- The average percentage of doing the textual tasks and the problems by the **students in the project schools** is 54%. The maximum results (8 points) of the project schools, in this area was achieved by 19 students (7%), and 10 students (4%) did not solve correctly neither of the tasks. The largest is the percentage of students (17%) who have a result of 5 points.
- **The students in the control schools** have an average percent of doing the test of 45%. All the tasks from this area have been solved correctly by 6 students (2 %), and no one of 14 students (5%) did solve a single task. In the control schools, the largest is the percentage of students (21%) who have scored 4 points.

- In the project schools, the average solving of the tasks in 2012 is for 24% higher than that in the measuring in 2009 (in 2012 – 58%, and in 2009 – 34%), and there is a csignificant improvement in doing all of the tasks.

The graph below shows the percents of correct responses of students to each one of the tasks in this area given in test, i.e. the results in 2012 and in 2009.



Graph 22. *Results of the students from the project schools in 2012 and in 2009 on Textual tasks and problems*

- The greatest increase (24%) between the two measurings of the achievements of students is noted in the selection of adequate table presentation of data given in a textual task and in the solving of a problem situation in two steps which includes the operations of adding and subtracting.
- The least increase in solving the task (15%) between the two measurings is noted in the task which in both studies was least solved – the open problem situation which could be solved in several ways: by drawing, graphically, on a table, by guessing nd checking, doing it from the back, by equation, and aliike.

Such task is given:

Task

Ивo, a Grade 7 student, sells used coloured pencils. Ivo sells 2 colored pencils for 3 denars.

✎✎ = 3 denars

If Ivo has earned 15 denars, how many coloured pencils has he sold?

Show how you did it:

Answer: _____ colored pencils.

In the table below are given descriptions of possible solutions and the percentage of students that have given: correct, partially correct, or incorrect response in 2009 and in 2012.

<table>
<tr><th></th><th>Solution</th><th>% in 2009</th><th>% in 2012</th></tr>
<tr><td>Correct</td><td>
By drawing :
3 + 3 + 3 + 3 + 3 = 15 denars
✎ + ✎ + ✎ + ✎ + ✎ = 10 coloured pencils
Arithmetical:
15 : 3 = 5 5 • 2 =10
Answer: 10 coloured pencils
By a table:
<table>
<tr><td>denars</td><td>coloured pencils</td></tr>
<tr><td>3</td><td>✎✎</td></tr>
<tr><td>6</td><td>✎✎✎✎</td></tr>
<tr><td>9</td><td>✎✎✎✎✎✎</td></tr>
<tr><td>12</td><td>✎✎✎✎✎✎✎✎</td></tr>
<tr><td>15</td><td>✎✎✎✎✎✎✎✎✎✎</td></tr>
</table>
Answer: 10
OR
<table>
<tr><td>denars</td><td>coloured pencils</td></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td>6</td><td>4</td></tr>
<tr><td>9</td><td>6</td></tr>
<tr><td>12</td><td>8</td></tr>
<tr><td>15</td><td>10</td></tr>
</table>
Answer: 10
<table>
<tr><td>denars</td><td>coloured pencils</td></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td>15</td><td>10</td></tr>
</table>
Algebraic:
If 2 coloured pencils cost 3 denars, one costs a denar and a half.. It means 15 denars, 10 coloured pencils
</td><td>5</td><td>16</td></tr>
<tr><td>Partially correct</td><td>Any of the mentioed steps, but the answer is 5 coloured pencils (2 coloured pencils treated as 1).</td><td>6</td><td>15</td></tr>
<tr><td>Incorrect</td><td>Any solution (numarical statement or procedure) which dont leads us to correct or partially correct answer.</td><td>89</td><td>69</td></tr>
</table>

In comparisson to 2009, where almost all the students who sold the task correctly or partially correctly – dit it arithmetically, in 2012 the task was solved in different ways, although there still dominate the arithmetical and the algebraic way of doing the task.

This task, but in different context was given also in the test for the teachers. In 2009, more than 2/3 of the teachers, as a correct way of doing the test accepted only the arithmetical way, and in 2012, between 79% and 83% of teachers accept as correct ones the other ways.

CONCLUSION

- The students from the project schools have solved the textual tasks and the problems signifivantly more successfully. The average percentage of doing these tasks well in 2012 by the students from the project schools is 54%, and by the students from the control schools it is e 45%.
- In 2012, in the project schools, there is a statistically significant improvement in the achievements of students, compared to the results of the students from the same schools in 2009.
- Although there an an increase in the achievements in this area, measured through the tasks on the test, the results are lower than the expectation prescribed by the curricula for the first cycle of the Primary Education.

5. ACCEPTANCE, IMPLEMENTATION AND NEEDS FOR FURTHER SUPPORT IN THE IMPLEMENTATION

The success of the changes in the instruction, depends, to a great extend on te concern of the teachers that are expected to implement a given innovation, as well as on the timely and adequate support that they are to receive for easier overcoming the concerns and moving on to the higher levels of implementation. Aiming to get more information that would enable better interpretation of the results, in the measuring in 2012, we included additional instruments that refer to the acceptance, implementation and the support to the teachers in the use of the principles and techniques from the *Thinking Mathematics* in the instruction in the early grades.

Below is given a short description of each of the instruments which were used in the second investigation, and the results from the investigation.

WAY OF MEASURING

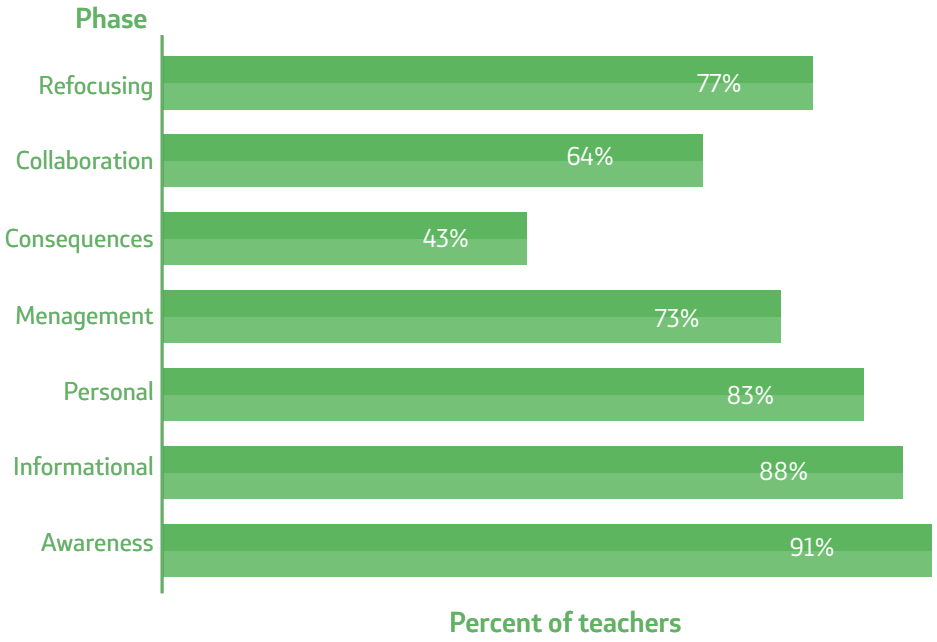
- The acceptance of the *Thinking Mathematics* and the concern of the teachers in its use was measured by a seven-level *Scale of concern* which contains 35 items.
- The level of use was measured by one question about the self-estimation of the level of the use of the principles and the approaches from *Thinking Mathematics,* as well as by one question about the frequency of their use in the instruction.
- The need of support was measured by one question about the needs of the teachers, with some aspects of the use of *Thinking Mathematics* that are important for its implementation in the everyday instruction.

5.1. Acceptance of Thinking Mathematics

Together with the introducing of *Thinking Mathematics in the early grades*, the teachers from the project schools were facing a process of change, which, according to certain investigations ((Hall (1977); Loucks&Hall (1979); Roberts (1993)), first

begins with the individual teacher and is of a very personal experience, and which imposes development of knowledge and skills, and together with it are included also the personal feelings/concerns. In measuring the level of acceptance and of concern in the implementation of *Thinking Mathematics* we used an addapted scale according to the Model of accepting changes accompanied with concerns (ConcernedBasedAdaptationModel – CBAM). In the analyses we related the results to the stages of concern (Stages of Concern – SOC), there are seven of them, and they are grouped in three categories of concern: i.e. concern about onself, concern about the tasks/activities and their implementation, as well concern about the effects. Theoretically, teachers are not faced with all types of concerns at the same time, but the concerns of one category are dominating, and when they are solved, dominating are the concerns of the next category. Hence, the support that is given to teachers should help them overcome the concerns they are facing with in that period, and on the contrary, it won't be very effective.

The graph below presents the percentage of teachers who have overcome each of the seven stages of concern. It can be noted that the majority of teachers, but not all of them, have overcome the concerns of each stage they were facing with in different periods of the three-year work with the Program *Thinking Mathematics*.



Graph 23. *Percentage of teachers from project schools who have overcome a particular stage of concern*

- In the first category – *the concern for oneself,* are those teachers who are in the stages: *awareness* for the *Thinking Mathematics* and its use, *the provided information* of the teacher and the training he/she has acquired, as well as the *personal concern,* i.e. for own coping with, in the use of the principles and techiques of *Thinking Mathematics*. This category of

concern, most oftenly appears, and is visible, at the beginning of the introducing of the implementation and refers to the need to have more information and resoponses to: confusion, dilemmas and questions.

- According to the responses of the teachers from the project schools, 91% of them have overcome the first stage and they are conscious about the Program and its needs.
- The stage of provided information (from the aspect of the experiences of teachers that have already been implementing the Program, and of the resources that they have at disposal, as well as of the implementation *Thinking Mathematics* that is different from the previous instruction), have been overcome by 88% of the teachers.
- The stage of *personal concern* refers to the issues related to the professional status and to the career developent of the teachers, the oportunity to pass decisions for the chages, as well as to the concern for the need of having more time, effort and devotion. This stage of concern was overcome by 83% of the teachers.

- The second category is *the concern for the tasks/activities* and their implementation and it contains elements from the stage *personal and concern about the organization of the instruction (management). This category appears at the beginning of the imlementation of the Project/ Program in the everyday work and in the activities of the teacher*, and it refers to the need for additional knowledge, skills and information for more efficient planning in the use of the activities (from the aspect of time, the needed materials, the organization of the work with students, including aims and activities of the Program in the year/thematic or daily planning and preparation of the instruction, and alike).
  - This stage was overcome by 73% of the teachers who are now sure about the activities and they do not represent to them a particular concern.
- The third category, *the concern for the project effects*, contains of the stages of concern for the *consequences, cooperation and sharing (collaboration)* and of the stage of *modification and refocusing* of the *Thinking Mathematics*. This category appears after the teachers have become more certain in planning, organization and use of the principles and techniques in their everyday practice. Here, the concern refers to the implementation of the newly acquired knowledge and skills and its influence upon the learning and the achievements of students, i.e. how to improve more the implementation, how much *Thinking Mahematics* is effective, compared to the previous way of work, how to cooperate and how to share the results with others, how can the Program be modified and be upbuild with an aim to improve it and to have a wider implementation of it (of other contents, topics and areas within the framework of Mathematics, as well as of other subjects).
- According to the responses, compared with the other two categories, i.e. the four previous stages of concerns, as it was expected, the percentage of

teachers who have subdued and overcome the concerns that belong to this category, is smaller. In accordance with the way in which the instrument is functioning, an anomaly is noticed – namely, it was expected that the percentage of teachers who have overcome the consequences concern will be higher than the percentage of teachers in collaboration and refocusing.

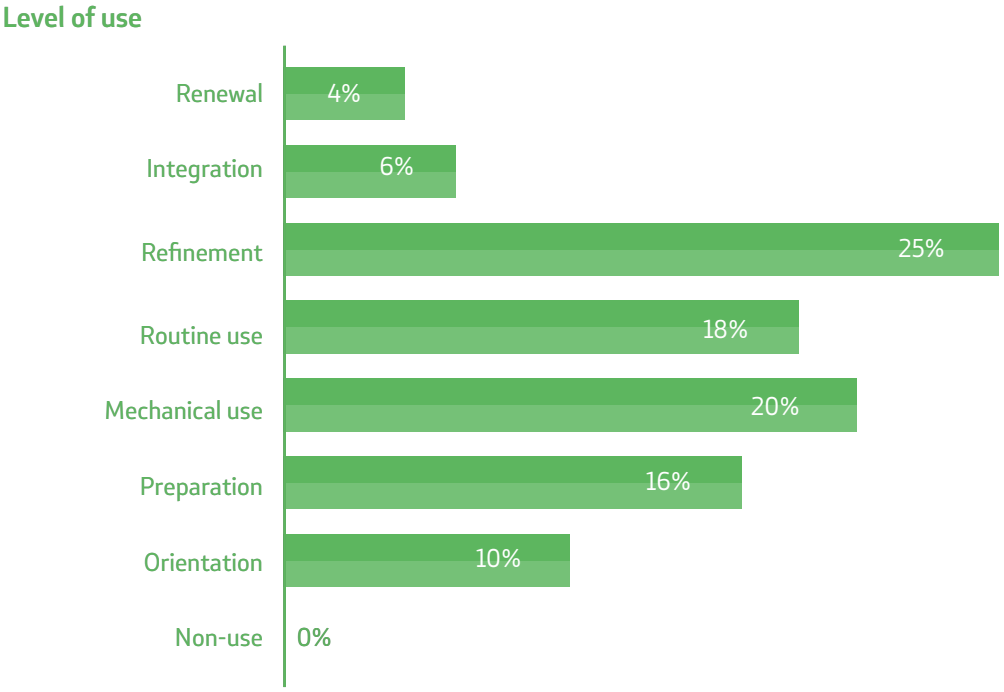
- It is obvious that, the smallest percentage of teachers (43%) have overcome the stage of consequences – i.e. the concern for the effects upon the students and their achievements, which theoretically preceeds to the concern of cooperation and sharing, and of modification and upbuilding. In relation to this, and also from other sources and investigations, we have notions that point out that at the level of the system in the country, more care is taken about what should the teacher do, than the influence of the teacher upon the students and the effects of what he is doing. Hence, probably, the teachers in the Project have devoted themselves more to dealing with the concerns related to the cooperation and to the sharing, and to the modification and the upbuilding of the Program.
- The concern for the cooperation and the sharing has been overcome by 64% of the teachers, but to a largest extent, the teachers do collaborate and share experiences in their own school and with the acting bodies of the teachers from the grade teaching phase, which is confirmed by the responses of the schools principals, the representatives from the pedagogues/psychologists, and of the teachers themselves on the question in the questionnaire, about how much they cooperate with other teachers.
- The concern for refocusing has been overcome by 77% of the teachers, but this most probably refers more to the selection and to the adaptation of the techniques that are being used, and are related to the implementation of the curricula at the classes in Mathematics. This refers to a small number of teachers concerning the knowledge and the use of other approaches in the teaching, and their relation to the new approaches in teaching which may be, could function better and could have greater effects and higher achievements with students, or to the adaptation based on evidence about the achevements of students.

5.2. Level of implementation of Thinking Mathematics

The responses to the question about the level of implementation of *Thinking Mathematics* were given by the teachers from the project schools, who have been trained, and who were expected to use the priciples and techniques of *Thinking Mathematics* three school years, i.e., with one generation of students in the first cycle of education (Grades 1 – 3). According to some investigations, the level of use of a given change/innovation in the instruction is related to the experiences of the teachers during the course of the use (Hall (1977); Loucks&Hall (1979); Roberts (1993). In most of the cases, on issues related to introducing a more extensive

Program, after Grade 3, they would be found on the average levels of use measured by the instrument for making self-estimates of the level of use (Levels of Use)[5].

The percentage of teachers, who in this investigation, have made a self-estimate that they belong to a particular level of use is given on the graph below.



Graph 24. *Percentage of teachers from project schools that are on each of the levels of implementation of the Program*

As it was expected, the largest percentage of teachers esteem themselves that they are at the average level of changes, i. e., at the level of mechanical use, the level of routine use or at the level of improvinig (refinement).

According to the descriptions for each level, given in the questionnaire, it can be noted that:

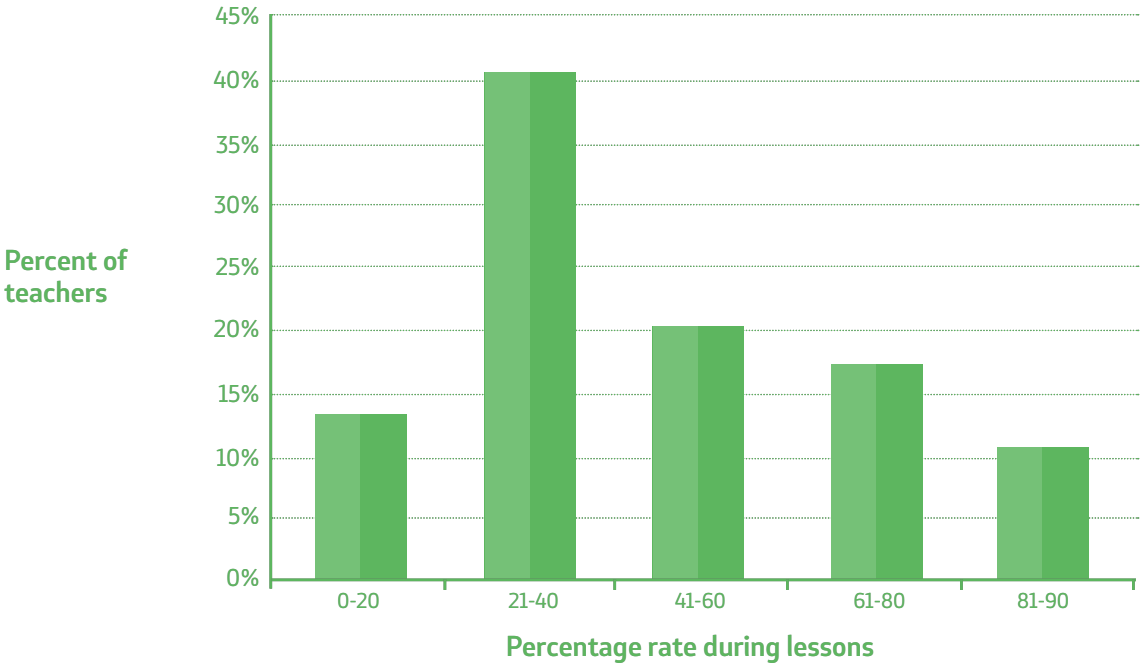
- 20% of the teachers (level of machanical use), are mainly directed to acquiring greater skills in using the principles and techniques of *Thinking Mathematics* and they devote little time to the reflexion;
- 18% teachers (level of routine use) feel sure and are satisfied with the use of the principles and techniques, and they use them as they were taught, and they consider making some changes in order to improve the use;
- 25% of teachers, who have made self-estimates that they are at the level of improvement, make variations of the use in order to increase the achievements, i.e., to have maximum effects with students.

5 Griffin, D. and Christensen, R. (1999).Concerns-Based Adoption Model (CBAM) Levels of Use of an Innovation (CBAM-LOU). Denton, Texas: Institute for the Integration of Technology into Teaching and Learning.

In addition to the fact, that all of the teachers who responded to the question that they are included in the implementation of the Program (0% – level of non-use), high is the percetage of teachers (10%) who are still reading references related to *Thinking Mathematics* and are preparing themselves to begin with its implementation, and high is also the percentage of teachers (16%) who are prepared for the implementation and have already planned and prepared the implementation.

The percentage of teachers with highest levels of implementation is the smallest one, 6% for integratiing, i.e. 4% for renewal (modifying). This means that few teachers are relating their activities from *Thinking Mathematics* to the activities of their coleagues with an aim to achieve general greater influence and effects, and still less they value the quality of the implementation and of the effects of the Program, in order to modify their own work and to developed something new. With regard to the above mentioned results, as well as to other investigations, it is realistic to expect that after three years of the implementation only a small number of teachers would achieve the highest levels.

Related to the implementation, in the following graph are given the results of the responses of the teachers to the question about the percentage of the instruction in Mathematics that they carry out using the principles of *Thinking Mathematics*.



Graph 25. *Aplication of the Thinking Mathematics in the teaching hours*

- The largest is the percentage of (40%) of teachers who stated that the approaches from *Thinking Mathematics* are being used on 21% to 40% in the class periods of Mathematics (that is 28% of the teachers in this category stated that they use them on 1/3 of their classes/lessons).

- Then 20% of the teachers stated that they use them from 41% to 60% of the lessons, and 17% of the teachers stated that they use them from 61% to 80% of their lessons.
- There are still teachers (13%) who use them very little, but there are, also, 10% of the teachers who stated that they use the principles of *Thinking Mathematics,* from 81% до 90% in their lessons of Mathematics (most probably this refers mainly to the use of the techniques of the Program).

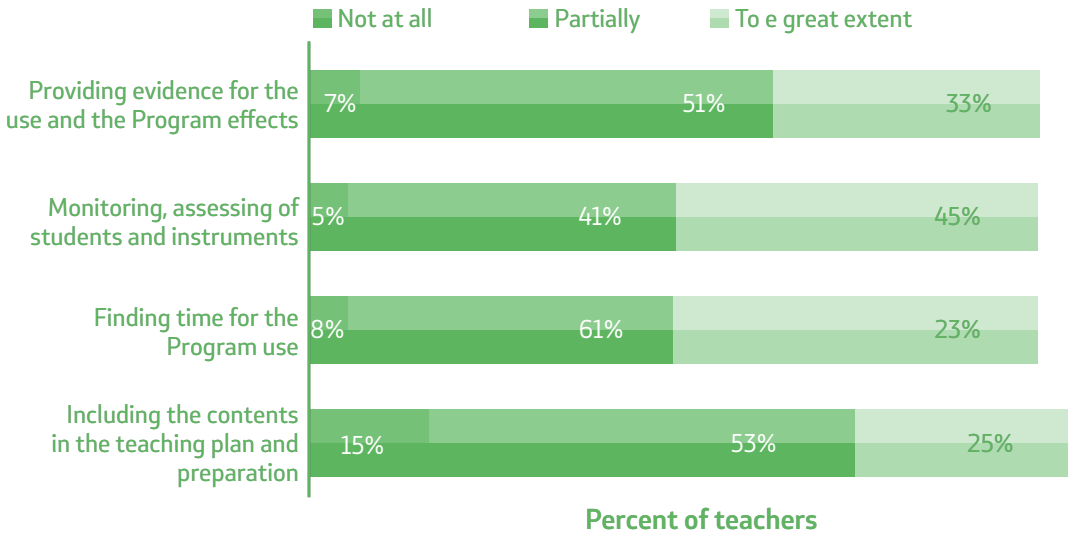
From the responses, it may be concluded that in practice, the majority of the teachers are still not directed to the instruction which includes the everyday principles of *Thinking Mathematics,* but to an instruction in which often used are the techniques that were part of the training, or of some other modified approaches.

Also, part of the school principals and the pedagogues/psychologists, in the interviewes about the imolementation responded that *Thinking Mathematics* is not implemented by all of the teachers.

5.3. The need of support for implementation

The teachers were asked about their needs for getting support in their use of *Thinking Mathematics* in everyday activities. Several aspects were offered, on which, the teachers, during the visits for support and for other issues, most oftenly, were pointing out that they need such support. For each of the aspects, the teachers were selecting the level of the needed support (not needed at all, partially nedeed, needed to a great extent).

Their responses are given on the graph below.



Graph 26. *Percentage[6] of teachers who need specifiic support*

The bigest is the percentage of teachers who need support related to providing evidence about the use and about the effects of that use (51% partially and 33% to a great extent), as well as in monitoring and assessing the achievements of students and in developing instruments for assessment (41% partially and 45% to a great extent). These two categories are related also to the measuring of the effects, and to this issue, relatively less time and attention was devoted during the training of the teachers for the Project. This is confirmed also by the test on pedagogical knowledge. Namely, in it, the teachers have shown least progress on the tasks that were related to the formative assessment, and the least number of teachers in estimating the stages of concern, have overcome the stage of concern about the consequences which is closely related to the competences for monitoring and measuring the effects of the Program.

Smaller is the percentage of teachers who, to a great extent need support in finding time for implementation (23%) and including the contents of the Program into the planning of instruction (25%), but in these categories, a support of a small amount is needed by 61%, i.e., by 53% of the teachers. In relation to the support of this kind, teachers should be more encouraged in order to be able more professionally to make use of the teaching time and to plan the teaching.

6 The total percantege is less than 100 due to the number of eachers who have not responded to the question.

CONCLUSION

- More than 2/3 of the teachers have overcome the concern related to the acceptance and use of *Thinking Mathematics*. But, still, there more than a half of the teachers who are concerned about how the use would influence upon the learning and the achievements og the students.
- The self-estimates about the level of use of the principles and the approaches of *Thinking Mathematics* have shown that a larger percentage of teachers are at the average lavels of use (mechanical use, routine use and improvement). According to the responses about its frequency, the teachers in more than 1/3 of their classes take care of and use the principles and the techinques.
- About 1/4 of the teachers, to a great extent, need support in including the principles and techiques from *Thinking Mathematics* in planning their use in the teaching, and between 1/3 and almost half of the teachers have a great need of support in assessing the students and in measuring the effects of the use.

PART II – INFORMATIONS ABOUT THE PROJECT

This part gives information about the beginning of the Project Thinking Mathematics in early grades as well as of the activities carried out so far.

A review is also given of the research findings of factors that influence the success of students, and that are related with the teachers who represented a starting-point in the investigation of the states, at the beginning of the implementation of the project activities (2009) and after three years from the implementation of the Project (2012).

1. BACKGROUND

Former Yugoslav Republic of Macedonia, introduced in 2007/2008 a nine year Primary Education, based on the Conception for nine year Primary Education and Enlightenment, adopted by the Minister of Education and Science.

Following the principles of Primary Education and Enlightenment, set up in the Cnception (first of all: the principal of general education character of the primary school, the principle of quality of education and the international comparisons of knowledge, the principle of active particicipation of students and the principle of the best interest for the child), the curricula for particular subjects in Primary Education were defined..

The curricula prescribe: the goals for a particular developmental period (Grades 1-3, 4-6, 7-9), the objectives per grades, the particular objectives concerning the selection of content, notions that are to be acquired and examples of activities and methods that teachers can use in achieving the set up objectives. The curricula, also, offer didactic guidelines for eachers, as well as general instructions for monitoring students' achievements.

In developing the curricula, consideration was also taken concerning the coverage of children in kindergarttens at the age of five (in 2006 it was lower than 20%) so that, in setting up the goals and the expected outcomes, some "lower expectations" were conscously supported in certain teaching subjects in the first developmental cycle (Grades 1-3) compared to those in other countries of the European Union, with an idea that together with the implementation of the curricula, there would immediately start with its ongoing evaluation, monitoring of students' achievements, as well as continuing work with teachers concerning the methods of work with students and in the way of reaching the goals and improving the understanding about the quality of education.

Parallel work will be done to achieve greater coverage of children in pre-school education and in introducing its compulsory status at the age of five.

1.1. Analyses of Curricula

The Bureau for Development of Education, having an interest in raising the quality of education, is striving continually to improve curricula, aiming to make students' achievements in the country be comparable to those of the students in other countries, and among other activities it supports carrying out projects focused on improving students' achievements in particular teaching subjects, including language literacy, numerical literacy and life skills.

Therefore, taking into consideration the results of TIMSS (Trends in International Mathematics and Science Study), which are among the lowest in Eastern Europe, there was an urgent need to make efforts to strengthen the capacities in the country to provide high quality instruction in Mathematics in primary education. The Bureau for Development of Education in cooperation with UNICEF Office – Skopje, began, in the course of 2008, implementing activities related to the component of numerical literacy aiming to develop curriculum for numerical literacy which would enable students' higher achievement, and which would give teachers opportunities to monitor students' development and to develop plans for achieving particular objectives and for the work in classes of Mathematics in Grades 1, 2 and 3.

The activities carried out during 2008, were led by international experts (Judy Rohde, M.A. and Eric Wilmot, Ph.D.) trough Consultancy Company Miske Witt & Associateds, and were focused on:

- making analysis of Mathematics curricula for Grades 1, 2, and 3;
- making comparative analysis of curricula in other countries, in order to review the corresponding content, expectations and organization of instruction;
- establishing broader aims (standards) in Mathematics;
- introducing the five topics (numbers and operations, algebra, geometry, measuring and analysis of data and probability); and
- establishing indicators for the expected and supposed skills for each grade related to each of the topics.

The analysis, show the curricula (for Grades 1, 2, and 3), compared to those in other countries, have generally low expectations[7].

The analysis does not focus only on the "weak" or on the "good" points, but, at the same time it gives recommendations to make more precise, to recompose and point out certain parts (aims, contents, didactical instructions, etc.) in the curricula.

The recommendations for all curricular topics refer, mainly, to bringing Mathematics closer to students, in an acceptable way, by using their experience and the already acquired knowledge. So, for ex. in acomplising the goals of ***getting sense of numbers***

7 For example, when students get acquainted with numbers in Grade 1, they are expected to be able to count up to 10. Then, in Grade 2, they expand their knowledge up to 20. In Grade 3, they work with numbers up to 100.

and operations it is recommended to enable students to use own strategies, to think about them and to share them with other students. It would help them not only to learn about their own thoughts and to learn from each other, but also, it would help the teachers to have insight into their process of thinking and to adapt their own performance of teaching.

In the part related to ***algebra and functions***, in order to be competitive at international level, it is recommended that students take part in activities which could improve their understanding about the models/patterns that are to be introduced and about the mathematical rules (for. ex. commutative, associative and distributive property) in solving problems.

The recommendation concerning studying ***geometry***, besides getting acquainted the students with plain geometrical forms and geometrical solids, icludes opportunities for students to describe the properties of forms they encounter. Also, students should be enabled to create symmetrical forms and be able to link notions of geometry with those of numbers and measurement.

In working with ***measuring*** it is recommended to establish relationship between content and other topics in Mathematics, and in doing ***data analysis*** it is recommended to introduce data collecting and analysis in Grade 1, so that students, at the end of Grade 2 and in the course of Grade 3, would start to develop the skill to make assumptions and conclusions based on data, as well as the skill to use the basic notions of probability.

It is pointed out in the analysis, that one of the strong sides of curricula is that the stress is put on using commercial and local materials. For ex. in Grade 3, the play monopoly refers to using commercial materials, and the environment in teaching geometrical forms and the use of bundles of sticks, as an illustration of adding and subtracting tens, refers to using local materials.

According to the analysis, the part about assessing students' achievements represents a promising framework for collecting and analyzing information about achievements and data about students progress, though it is necessary to state more precise standards and indicators for measuring achievement in order to make it possible to assess the achievement of each individual student.

1.2. Training of trainers

On the basis of the considerations from the analysis, an expert team developed a program to train national trainers, aiming to train teachers to implement the given recommendations related to curricula in their instruction. In developing and implementing the entire training, the materials from "Thinking Mathematics for Grades 1, 2 and 3" were used – a program for training teachers (based on conducted international studies) prepared by the American Federation of Teachers, one of the two larger federation in U.S.A. In addition, Dr. William Schmidt, the director of the TIMSS project in U.S.A, from Michigan State University was consulted, who confirmed that "Thinking Mathematics" is a program that enables teachers to acquire profound knowledge which would change the instruction and enable their students to improve the performance. So, in the state of Minnesota, in U.S.A., the development of adequate standards in the Mathematics curricula and the intensive training of teachers using the program "Thinking Mathematics in Grades 1, 2, and 3" and other similar materials, resulted in improving students' achievement in Mathematics, in the period 1995-2007, within the framework of the TIMSS study.

The American Association of Teachers enabled the program "Thinking Mathematics in Grades 1, 2 and 3", to be used freely and without charge. The preparation of materials for mathematics was done by Ms. Judy Rohde, consultant, and by Ms. Alice Gill, the author of "Thinking Mathematics".

The Bureau for Development of Education, in order to include better quality teachers, i.e. highly motivated teachers, to improve mathematics instruction, on the basis of open competition, made a selection of about fifty teachers (part of whose language of instruction is Macedonian, Albanian, Turkish or Serbian) and required from them:

- to attend the training structured in a total of 3 workshops;
- to implement the acquired knowledge in performing the instruction (with support of BDE advisers) and
- to train all teachers teaching mathematics in Grades 1, 2, and 3 (first as trainers in their own schools providing training to teachers teaching in Grades 1, 2, and 3, as recommended by the advisers and the international team of trainers).

The main objectives of the training in *numerical literacy* are:

- to deepen the understanding about the quality of mathematics instruction by teachers;
- to implement the newly acquired knowledge in the instruction and so to become more efficient teachers;
- to improve students' achievements in the mathematics teaching subject;
- to improve students' results in mathematics within the framework of the international comparative investigations and studies

Workshop Session One took place from 09.03 to 14.03. 2009, and it consisted of five-day training led by Ms. Judy Rohde, senior consultant, in cooperation with Ms. Mary Ellen Knappmiller and Ms. Marium Toure. The participants at the workshop were acquainted with the framework and the Ten Principles of Thinking Mathematics, and how they are used to facilitate the development path of students from counting to addition and subtraction.

Workshop Session Two took place from 15.05 to 20.05. 2009 and it consisted, also, of five day training led by the same consultants. The workshop dealt with the strategies that could be used to solve addition and subtraction textual problems.

Workshop Session Three took place from 22.06 to 27.06. 2009. The training was led by the same trainers. The topics of the workshop dealt with taking evidence (recording), questioning and assessing students, multiplication and division, data processing, problem situations.

1.3. Dissemination of training, monitoring the implementation and support

After the training of the teachers – future trainers and of the advisers from the Bureau for Development of Education, a phase of dissemination followed, first in the schools from which the trained teachers came from. The teachers, under the mentorship of the advisers, were implementing the acquired knowledge from the semnars at their Mathematics'classes, and then provided training for all the teachers that were teaching in Grades 1 to 3 in their own schools (phase 1) at the end of 2009 and at the beginning of 2010. After this, the teachers trainers (a total of 46 teachers) acquired certificates for main trainers and could provide training for the teachers from the other schools.

In the next phase (phase 2), which took plce in the period May-June 2010, each main trainer organized training for all grade teachers in one school that located close to the home school of the trainer (46 schools).

In phase 3, 60 schools were included, so that care was taken to include schools from regions which had not been previously included in the training. In each of those schools, first two teachers were trained for schools trainers, who with the support of the advisers from the BDE, provided dissemination of the training with the teachers in their own shools in the first half-term of the school year 2011/2012.

Phase 4, began in the second half-term of the school year 2011/2012, when school trainers from 64 primary schools were trained. It is planned that these trainers will be engaged to train the teachers from their own respective schools in the first half-term of 2013.

With regard to the trainings, it should be pointed out that, although at the beginning it was not planned, the trainings included all the teachers from grade

teaching and the representatives from pedagugues/psychiologist in the schools, and in some schools were also included the school principals, in order to be able more adequately to give support to teachers, to monitor the imlementation of the instruction and be included in the process of certification.

The implementation of the approaches from *Thinking Mahematics,* both from the aspect of the quality of the implemented instruction, as well as of from the aspect of the results achieved by students, was monitored by the Bureau for the Development of Education and by the main trainers. The aim is to give direct support to teachers in their school, and so far, at least two visits have been organized to give support to each school from phase 2, and one visit to each school from phase 3, and in the schols from phase 1 it is expected thet the main trainers have given continuing support to their colleagues. Within the framework of the Project, with an aim that the main trainers give more qualitative professional and pedagogical support, in October 2011, the national trainers were attending a two-day training for mentorship support, and in November 2012 they were included in a training for recording good practices of the Project *Thinking Mathematics in the early grades.*

Also, at the beginning of 2012, the criteria for certification of the teachers which have shown the implementation of the acquired knowledge in skills in their everyday practice were developed. The school teams and the advisers from the BDE are rsponsible for the certification. According to the reports received by the advisers, there are teachers, and especially from the schools from phase 1and from phase 2, who have meet the criteria and have successfully gone through the process of certfication.

2. FINDINGS FROM PRIOR INVETSIGATIONS

2.1. Investigations about some relevant factors of students' achievements related to teachers

In undertaking educational reforms and in monitoring their effects, care is taken of factors having impact on the effects, and especially on those on which it could be influenced. Here, in addition to curricula and school facilities, the teacher has a central place. Hence, in monitoring the effects of the project *Thinking Mathematics*, in addition to the program for new approaches in learning Mathematics, based on the Ten Principles, care should be taken on other factors related to the teacher which could have impact upon the effects of the program. A review of investigations, related to certain characteristics of teachers that might be relevant in conducting and monitoring the activities of Thinking Mathematics, grouped mainly according to Koehler and Grouws' model (1992), and revised by Suriza van der Sandt (2007), is given below.

2.1.1. Teachers' knowledge of Mathematics

Investigations related to teacher's knowledge refer, most often, to:

1. Content (mathematical) knowledge and to
2. Pedagogical knowledge (knowledge how to teach Mathematics).

Findings from investigations show that:

- The way how the teacher performs the instruction., and its effects upon students' achievements, depend on how much teachers know the mathematical contents that they are teaching (Muijs & Reynolds, 2002; Ball & Bass, 2000, according to Van der Sandt S. (2007)). In 16 out of 18 analyzed studies, it is stated that there is a direct relationship between the content and the teaching practice (Horison Research (2008)). In additions to the methodological limitations, in the majority of these studies, the one-way findings give ground to make generalizations.
- The broadening of Mathematics knowledge of teachers can bring a change of the way of instruction and to deeper connection with THE pedagogical knowledge (Ormrod and Cole (1996) according to Van der Sandt S. (2007)).
- Teacher's conduct in teaching depends on teacher's understanding of the

content that he/she is teaching, and of his/her understanding how students learn. (National Research Council (NRC, 2001), Van der Sandt S. (2007)).

- Knowledge and beliefs of teachers are related and have impact on students' achievement (Muijs and Reynolds (2002) according to Van der Sandt S. (2007)).
- One of the important factors in changing teaching practice, which should be taken into consideration, is the knowledge of teachers (Clarke (1997) according to Bergeson T. (2000)).
- Pedagogical mathematics knowledge of teachers (measured directly, by a knowledge test) is directly related to students achievements in Grades 1, 2, and 3. (Hill, Rowan & Ball (2005)).
- Mathematics knowledge is one the 12 factors related to the teacher which have impact on students' achievements (Schacter & Thum, (2004), според Goe L, L. Sticker (2008)).

2.1.2. Attitudes and beliefs of teachers

Most often investigated attitudes of teachers, are those related to Mathematics and to the beliefs how Mathematics should be taught. Research findings show that:

- Teachers' beliefs related to the nature of Mathematics and to the general concepts how to teach Mathematics, have strong impact on the teaching, due to the fact that on the basis of the beliefs teachers decide what they are going to teach, to what part they are going to give bigger importance, how to teach and how they would behave towards student's learning (NRC, (2001); Muijs & Reynolds, (2002); Schoenfeld, (2001) according to Van der Sandt S. (2007), Handal & Herrington, (2003); Kagan, (1992); Pajares, (1992) according to Yates S. (2006))
- Teachers' beliefs are often a filter for the new knowledge and they can speed up or make slower the reform undertakings (Burkhardt, Fraser &Ridgway, (1990); Koehler & Grouws, (1992); Sosniak, Ethington & Varelas, (1991), according to Yates S. (2006)).
- Each change in the teaching of Mathematics should take into consideration teachers' beliefs and their changes (Swan (2006) according to Clarke J. (2008)).
- Attitudes towards Mathematics and to the teaching of Mathematics have impact upon formation of students'attitudes towards Mathematics, and via that to their achievements in Mathematics (Ernest, (1989) according to Van der Sandt S. (2007)).
- There is a positive relationship between the degree of cognitiv-constructivistic orientation in pedagogical beliefs of teachers and students' achievements in solving problem tasks in Grade 1 (Peterson, Fennema, et al. (1989) and Staub & Stern, (2002)).

2.1.3. Attitudes and concerns of teachers about the changes

The success of the implementation in the instruction, depends to a great extent, on the concern of the teachers who are expected to implement a given innovation, as well as on the timely and the adequate support that they should receive to overcome easily the concerns, and to proceed to the higher levels of application. In most of the investigation that have been done, in this relation the Model of accepting changes accompanied by the concerns (Concernes Based Adaptation Model – CBAM) or more precisely the stages of concern (Stages of Concern) that refer to the affective side of the implementation (reactions, feelings and perceptions) of the teachers and the levels of use (Levels of Use) related to the behaviors and to the presentation of the teacher to a given change.

Findings from investigations[8] show that:

- the implementation of cooperative learning techniques have improved after the given support to the teachers corresponding to their concerns (Anderson, Rolheiser, & Bennet, (1995); Hargreaves at al. (2002); Cheuning &Yip (2004));
- the addaptation of the training, taking into consideration the concerns of the teachers, results in a better implementation of the innovations and a bigger focus towards the students (Casey & Rakes (2002));
- the support in the understanding and in the implementation of an innovation to teachers, improves the learning of students (Borner (2003); Bennett, Fullan, & Rolheiser, eds. (2006));
- the level of the implementation is related to the experience of the teachers and the length of time of the implementation of the innovation (Hall (1977); Loucks & Hall (1979); Roberts (1993));
- the skills of the trainers, the support from the school management, the condition with the resources for learning and other materials, the commitment of the teachers, the school ethos, have influene upon the level of the implementation of the innovation (Loucks & Melle (1980); Hall, Hord, & Griffin) (1980); Robers (1993); Krasner (1999)).

8 Archie A. George, Gene E. Hall, Suzanne M. Stiegelbauer (2008): Measuring implementation in Schools: The Stages of Concern Questionaire, SEDL, pg. 66, and Archie A. George, Gene E. Hall, Suzanne M. Stiegelbauer (2008): Measuring implementation in Schools: The Stages of Concern Questionaire & Levels of Use, SEDL, pg. 30.

PART III – METHODOLOGY OF THE INVESTIGATION

This part gives a brief description of the methodology used in the baseline study, i.e. the information about the aims of the study, the conceptual framework, the indicators for the study, the used instruments, sample, and the collecting, processing and analysis of data

In order to monitor the quality of the application of the approaches in numerical literacy in Thinking Mahematics in the early grades, as well as of the results achieved by students, two investigations have been undertaken (in November 2009 and in November 2012). Prior to the beginning of the project activities, the initial state was examined (the baseline investigation), and the investigation was repeated after three years when those teachers taught by trained teachers that had been included in the first phase of the Program, completed the first cycle of the Primary Education (from Grade I to Grade III).

The used methodological approach was based upon the following principles:

1. Focus on providing valid and reliable information that are necessary for evaluation of the goals of the Project and the influence upon the Project activities;
2. Providing data that may be used in explaining the states;
3. Providing adequate data for drawing conclusions and decisions, and in planning further project activities;
4. Rationality from the aspect of the time line, the included human resources and means.

In the investigation, a quantitative and qualitative approach was used.

1. OBJECTIVES OF THE INVESTIGATION

The objective of this investigation was to provide relevant information about the state at the end of the first cycle in the implementation of the Project, and on the basis of their comparisson with the investigation of the initial state (conducted in 2009), to draw conclusions about the level of achevement of the goal of the Project (*to raise the level of knowledge and skills of the teachers and to improve the achievements of students in Mathematics)*.

Particular aims of this investigation were:

1. To provide information about the **changes with the trained teachers** related to:
   - their attitudes to teaching and learning Mathematics (pedagogy in the instruction of Mathematics based on the preinciples of the Program *Thinking* Mathematics and
   - their mathematical and pedagogical knowledge that are relevant for performing the instruction in *Thinking Mathematics.*
2. To provide information about the **differences in the achievements** at the end of Grade III between the students taught by the teachers trained to apply the principles and techniques from *Thinking Mathematics* (students from the project schools), and the students that were taught by teachers who have not been trained to implement the Program (students from the control schools).
3. To provide information about the implementation of *Thinking* Mathematics in the project schools, the concers of the teachers and the support they have received.
4. To give directions for the future activities of the Project.

2. CONCEPTUAL FRAMEWORK

In order to make an esrimate of the effects from *Thinking Mathematics* upon the teachers in their instruction of Mathematics and upon the achievements of students in Mathematics, the following two common ways of measuring the changes were used:

1. *„with and without the activiities "* – this is an quasi-experimental approach where, in one group of schools (projects schools) are conduced the activities (the teachers are trained and the Project is implemented), and in the other group (control schools) no changes are made in relation to the usual way of instruction (the teachers are not trained) and
2. *„before and after the activities "* – where the changes in the project schools are measured during the course of a determined time period.

In the investigations of the initial state and in the consequitive investigation, and on the basis of the review of the investigations about the factors that influence considerably to the achievements of students, we decided to investigate the states and the changes both in the project and in the control schools in relation sto:

Factors related to teacher (FT)

- Mathematical knowledge
- Pedagogical knowledge for teaching Mathematics
- Attitudes towards Mathematics and to the teaching of Mathematics *[9]
- Attitudes towards learning Mathematics and to the pedagogical approaches in teaching Mathematics
- Expectations from students
- Familiarity with curricula
- Level of training for the application of interactive methods in the instruction (instruction focused on the student)*
- Level of using and grades of concern in the implementation of the Project **

Factors related to socio-economic environment (FSE)

- Parental education *

9 Factors marked with * were investigated only in the Baseline Study (in 2009) in order to determine whether the project and the control schools are even and upon them no intervetion have been made that might bring changes. Factors marked with ** were examined in the consecutive measuring (2012) and are relatd to the experiences from the implementation of Thinking Mathematics and may considerably influence upon its' effects.

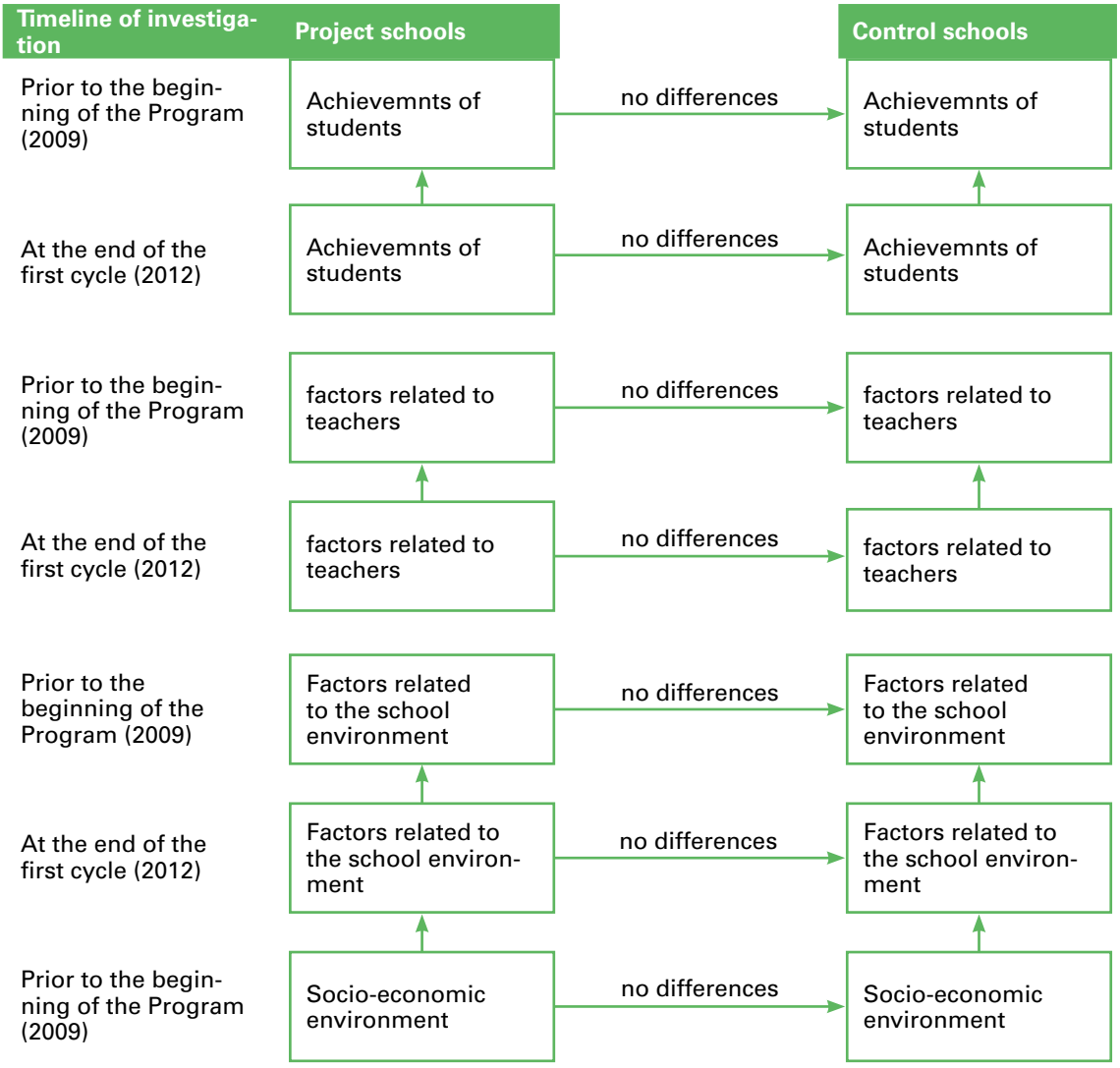
Factors related to school environment (FSE)

- Mutual support of the school staff
- School facilities for the instruction of Mathematics

Factors related to the support by the school management staff (FMS)

- Considerations about the current states of the achievements in Mathematics **
- Activities for improving the instruction of Mathematics **
- Support to teachers in the implementation of *Thinking Mathematics* **

In accordance with the conceptual framework, developed at the beginning of the investigation (Supplement 1) below is presented the way in which the changes in the examined factors in the project and in the control schols were compared.



The factors related to the support by the school management staff, the concern of the teachers for the implementation of the Project, the estimate of the level of using the new approaches and the support that is needed by the teachers, were measured only in the project schools, and in the largest part only at the end of the first cycle of the implementation.

3. INDICATORS

The indicators, given below, were defined especially for this investigation on the basis of the Project they and were measured in the investigation. The results presented in Part 1 of this Report, are given acoording to the below defined indicators.

I. 1 – Teachers' understanding of learning and teaching students and the compatibility with the way of teaching based on the Ten Principles in learning *Thinking Mathematics* (understanding of teaching and learning of Mathematics)[10]

- Attitudes towards learnig Mathematics and to the teaching based on the Ten Principles of *Thinking Mathematics.*
- Pedagogical knowledge of teachers in accordance with the approach in *Thniking Mathematics.*
- Teachers' expectations concerning the achievements of Mathematics of their students.
- Familiarity with Mathematics curricula for the susequent cycles in Prmary Education.

I.2 – Teachers' knowledge and understanding of mathematical concepts that are crucial in *Thinking Mathematics* (maths' knowledge of teachers)

Maths' knowlege of teachers and their understanding of:

- the concept of number;
- the four basic arithmetical operations and their properties;
- the story tasks and the solving of problems.

I.3 – Support to the teaching of Mathematics by the school management staff (Support to the changes in the teaching of Mathematics)

1. To what extent the representatives from the school management staff:

- are satisfied with the achievements in Mathematics in the grade teaching cycle;

10 The indicator is given as formulated in the Logical framework of the investigation (Supplement 1), and the shortened names that are later used are given in brackets.

- undertake activities in promoting the teaching of Mathematics.

2. Support to the teachers in the instruction of Mathematics:

- cooperation with other teachers;
- support to the changes that they are introducing.

3. Activities for improvement the instruction of Mathematics

- implementation of the Project activities in school;
- other activities in improving the instruction of Mathematics.

4. Equipment with manipulative/handy aids for the instruction of Mathematics.

I. 4 – Achievements of students

Students' achievements on the Mathematics' test, which contained tasks that measure the conceptual and procedural knowledge, understanding and using natural numbers, the four basic operations, as well as solving story tasks and problems.

I. 5 – Acceptance, implementation and the need for further support to the *Thinking Mathematics* (Acceptance, implementation and support) [11]

- level of implementation of the Program;
- level of concerns for the Program;
- need for support in various areas.

11 This indicator was used only in the project schools, in the measuring at the end of the first cycle because it reders to experiences, concerns and the needs related to the Project.

4. METHODS AND INSTRUMENTS FOR COLLECTING DATA

In accordance with the defined indicators, the following sources and methods for collecting data were used in this investigation:

Investigation of teachers

By using specially prepared tasks, questions, scales of attitudes and vignettes on teaching situations, we collected data about the current mathematical and pedagogical knowledge of teachers to teach Mathematics, as well as data about the atitudes to learning and teaching Mathematics, the expectations from their students, familiarity with curricula, and by the trained teachers, also, we collected their estimates about the level of the implementation and the needs for support.

Assessing maths' knowledge and understanding of students

By using a specially developed test, administered at the beginning of Grade 4, we collected data about the knowledge of students acquired at the end of Grade 3 in the areas: numbers and the four basic mathematical operations, as well as in solving textual tasks and simple problems.

Interview with school principals and pedagogues/psychologists

By using a semi-structured interview, we collected information on the instruction of Mathematics, on the support to teachers in the implementation of the instruction, on the equipment in the teaching of Mathematics, as well as on their personal engagement in the project *Thinking Mathematics*.

The majority of the collected data were qualitative, in order to provide objective comparissons between the project and the control schools prior to the beginning of the project activities and at the end of the first cycle in the implementation of the Project.

A brief description of the content of each of the used instruments is given bellow. The instruments for the investigation in the project and in the control schools, to a largest part, were the same in order to enable that comparissons is more objective for a wider circle of the users of this Report.

The psychometric characteristics of the instruments used in the investigation at the end of the first cycle are given in Supplement 3.

Instrument	Brief description
Questionnaire for teachers[2]	The questionnaire has four parts: **The first part contains:** ▸ 5-level *Scale of attitudes related to the learning of Mathematics and the pedagogical approaches in teaching Mathematics* based on the 10 principals of *Thinking Mathematics;* ▸ 4 questions related to various aspects of the instruction in Mathematics, the expectations of the teachers about the knowledge of students at the end of Grade 3, the equipment of classrooms for the instruction of Mathemathics, participation at trainings and in projects for the teaching of Mathematics. **The second part contains:** ▸ *Test on Mathematics knowledge of teachers,* which contains 15 items, where, in order to give responses, the teachers were to use their general Mathematics' knowledge, and the Mathematics' knowledge that is important for the teaching Mathematics in Grades 1 through 3; ▸ *Test on pedagogical knowledge of teachers* which contains 11 teaching situations, where, in order to give responses, the teachers are to use their pedagogical knowledge. **The third part contains:** ▸ A seven level *Scale of concern about using the approaches from Thinking Mathematics*, which contains 35 items; ▸ Self-estimate of the level of using the principles and approaches from *Thinking Mathematics*; ▸ two questions on the frequency of using the *Thinking Mathematics* and the need to give support. **The fourth part contains** 3 questions related to the demographic data about the teachers.
Test for students	*Te test forthe students* has a total of 19 tasks (21 items) which measure the knowledge and skills in the areas: ▸ *Number concept* – 5 items; ▸ *Opperations and characteristics of operations* – 11 items and ▸ *Problem situations* – 4 items. 9 multiple-choice items, 6 items with brief short answer, and 4 open-ended items, were used, which required the complete procedure in giving responses.
Protocol for the interview[13]	Two groups of questions were given in the reminder for the interviews with the school management staff: ▸ A group of 3 questions related to the equipment for the instruction and for the cooperation between the teachers, and ▸ A group of 6 questions related to the experiences in the implementation of *Thinking Mathematics.*

12 The questionnaire for the teachers in the project schools and in the control schools differed in the third part which was designed only for the questionnaire for the teachers from the project schools..

13 The Protocol for the Interview was used only in the project schools.

Before the use of the instruments in the baseline investigation of the Program, the psychometric characteristics of the scales and of the tests were checked. The tasks/items that were not discriminative were not taken into consideration in processing the data in both measurings.

5. SAMPLE

5.1. Selection of sample

One of the research challenges was to identify two samples – one for the project schools and one for the control schools. The population of the project schools consisted of 35 schools which took part in the Project upon applying to an open competition for teachers – future trainers in the Project. It shows that the project schools are not representative for the entire population of the schools in the country, though by the end of the Project, it has an objective to include all the schools. We had a dilemma to measure the beginning state with a representative sample of schools, teachers and students at national level, and in that way to provide a highly reliable basis for comparing the effects when all the schools would be included, or to develop a sample comparative (parallel) to the starting Project sample, which would provide more reliable comparing of the effects during the course of the Project, and indicators for intervention during its implementation, and we decided to choose the second approach. So, two comparable cluster samples were selected:

1. Sample of project schools and
2. Sample of the control schools.

In selecting the sample of project schools, care was tahen that it should, as much as possible to represent the population of the primary schools in country in relation to:

- geographical distribution;
- location of schools (urban – rural) and
- language of instruction (Macedonian or Albanian).

15 project schools were selected, but due to the limited number of project schools and their characteristics concerning the mentioned factors (the larger part are urban and with instruction in Macedonian language) their representation in relation to all of the schools in the country, should be considered with a reserve.

Then, we deliberately made a selection of parallel schools, taking into consideration that they be identical concerning the geographical coverage (from the same towns and similar villages) and the language of instruction, and similar concerning

the social background (which was later checked through the education of their parents) with that of the project schools. The investigation in 2009 was conducted in those schools.

After the beginning of the *Thinking Mathematics,* and in the planing of the investigation for 2012, it was stated that there will be small changes in the sample:

- in one of the project school in the sample of 2009, the training was not carried out because the trainer had left the school. Therefore, in the repeated investigation this school and is corresponding control school, were exculded;
- in an other school, because of the instructions to train only the teachers from the first cycle, there were no trained teachers who teach in Grade 4 classes. (i.e. students that from Grade 1 to Grade 3 were taught by a teacher who had been trained for the *Thinking Mathematics).* This school was repleced with the closest school with similar characteristics in which all of the teachers had completed their training three months after the trainings in the original project school, and the trainer was the same one as that in the project school; and
- due to the fact that in one of the bilingual schools the training was carried out only for the teachers doing the instruction in Albanian language, the investigation for the instruction in Macedonian language was conducted in the closest bilingual school. Replacement school was with similar characteristics, the teachers that doing the instruction in Macedonian languages have undergone the training three months after the training in the project school, and the trainer was the same one as that in the original project school.

The sample of the schools is given in Supplement 2, and the characteristics of the sample are given in the following table:

Table 5. *Sample of schools according to urban/rural coverage and according to language of instruction*

Geographical coverage	Project schools			Control schools			Total
	Macedonian	Albanian	Mixed	Macedonian	Albanian	Mixed	
Urban	9	1	2	9	1	2	24
Rural		1[14] (2)[15]	1		1 (2)	1	4 (6)
Total	9	3	3	9	3	3	28 (30)

14 The investigation in 2012 was conducted in one project and in one control school in this category, and the total number of schools is 28.

15 The numbers in brackets refer to the measuring in 2009.

5.2. Selection of students

The population of students involved in measuring the effects f the Project, are those students from the project and from the control schools that have completed Grade 3, so that the Project, by its content was focused on the first cycle of Primary Education (Grades 1- 3). The investigation prior to the beginnng of the Project, and at the end of the first cycle in its implementation, was conducted at the beginning of the school year with studets from Grade 4[16] which represented a population closest to the target group.

In the selected project and control schools prior to the beginning of the Project, and in the control schools in the investigation at the end of the first cycle, 20 students from Grade 4, were selected by random sample. In the Project schools,a in the investigation at the end of the first cycle of the implementation of the Project, the selection was done only from students taught by teachers trained in the principles and techinques of *Thinking Mathematics.* The investigation in 2012 was conducted on a total of 557 students, and that one in 2009 was with 598 students.

5.3. Selection of teachers

In each of the schools, 10 grade teachers from the project schools and from the control schools were selected by random sample for the investigation in 2009, and for the investigation in 2012. In the project schools the selection for the investigation in 2012 was made only of the teachers that were trained in the Project *Thinking Mathematics in the early grades.*

The investigation in 2012 was conducted with 276 teachers (with 138 teachers in project schools and with the same number i the control schools), and in 2009 it was conducted on a total of 299 teachers.

The following table presents their gender, education and age structure.

16 At the beginning of Grade 4, the students have already studied the contents of Mathematics for which they were tested, and this was in accordance with the content of the trainings in the Thinking Mathematics.

Table 6: *Characteristics of the sample of teachers in 2009 and in 2012*

Characteristic		Project schools				Control schools			
		in 2009		in 2012		in 2009		in 2012	
		number	%[17]	number	%	number	%	number	%
Gender	Female	113	75	129	94	116	78	111	80
	Male	22	15	9	6	20	13	13	9
Education	Higher – pre-school[18]	/	/	7	5	/	/	5	4
	Higher – grade teaching	40	27	34	25	37	25	30	22
	University – grade teaching	78	52	75	54	71	48	70	51
	University - pedagogy	16	11	19	14	27	18	15	11
Working experience	Up to 5 years	9	13	5	4	8	6	14	11
	5 – 10 years	20	15	21	16	10	7	12	10
	11 – 20 years	46	34	39	29	66	49	37	30
	Over 20 years	60	28	68	51	50	27	60	48

According to the controlled characteristics, there were no considerable differences in both measurings between the project and the control schools concerning the sex structure, education structure, as well as concerning the working experience.

5.4. Selection of school principals and pedagogues/psychologists

The sample of the school management staff consisted of school principals and one pedagogue or psychologist from the project schools. However, due to the great interest and the readiness to take part in the interview with the members of the school management teams, the implemented sample in the investigation of 2009, is a little larger (15 school principals and 2 deputy school principals, 8 psychologists and 11 pedagogues).

In the investigation of 2012, the interviewed were:

- 16 school principals;
- 3 psychologists;
- 12 pedagogues.

In case when in the school, there were two pedagogues/psychologists, we interviewed the one who had been more included in the project activities. Seven of the iterviewed school principals or pedagogues/psychologists were trained within the *Thinking Mathematics*.

17 The remaining part to 100% are those that have not responded to the corresponded question.

18 In 2009 година, this profile of teachers was not working with students that were included in the sample.

6. DATA COLLECTION, DATA PROCESSING AND ANALYSIS

Collection of data was conducted by selected researchers who had had previous experience in conducting simlar investigations. At first, they went through an organized one-day training, and then were set up the time limits for the tests for students and the questionnaires, as well as for the interviews with the school principal and the pedagogue/psycologist.

The collection of data began on October 30, and ended on November 2012.. Then a coding of the filled in instruments, reviewing of the tests for teachers and of the test for students, data entry and qualitative processing of the evidented responses of the interviews, were done

Dates were entered into Excel programme, and they were processed by using:

- TIA Plus programme (for the scale and for the tests for teachers and for the tests for the students), by which the psychometric characteristics of the instruments were checked, the results of the tests and the scales of attitudes were estimated and comparisons were made between the project and the control schools in relation to the results on the tests and to the scales of attitudes, as well as comparisons of the results between the first and the second investigation).
- SPSS programme used in processing the responses to the questions and in making comparisons of the responses to the questions between the project and the control schools, as well as in the comparisons between the first and the second investigation.
- SOCQ 075 Graph and Print and Excel that was used in processing the data of the *Scale of concerns for the Program*.

The quantitative and qualitative data were analyzed in relation to the defined indicators by using topic analisis.

The analyses were made as it is shown in the conceptual framework (Part 2, heading 2 of this Report). The differences between the project and the control schools were of primary importrance. In case they were statistically significant, we did not proceed to other analyses. When the differences were not significant, we made additional comparisons between the project and the control schools, aiming to discover the factors that would eventaually influence upon the project schools not to achieve the presupposed and the desired effects. In the cases when the control schools have also shown an increase in the achievements, efforts were made to find out which other activities have happened in the schools, which could have an influence upon the achievements.

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SUPPLEMENTS

1. **Logical framework of the investigation**
2. **List of schools in the sample**
3. **Measurable characteristics of the instruments**

SUPPLEMET 1.

PROJECT: Thinking Mathematics in the Early Grades

INDICATORS

Aim	Tasks	Activities	Baseline Indicators	Outcome Indicators	Impact Indicators	Methods/ instruments
Build the capacities of teachers in the instruction of Matematics for improving the achievemen-ts of students	Developing capacities (preparation of trainers) in the training of teachers from the grade teaching phase for using new approaches in the teaching of Mathematics Training of teachers from the grade teaching phase for using new approaches in the teaching of Mathematics	Training of teachers for the Thinking Mathematics Trainins of teachers		Number of trained trainers Number of trained trainers		
	Teachers are expected to use the Ten Principles and techniques from Thinking Mathematics in the instruction		How close is the present teaching practice in the teaching of Mathematics, with the instruction base on the Ten Principles of Thinking Mathematics • The average value of acceptance of the items on the Linkert's scale which indicates instruction based on the Ten Principles. • % of trained teachers whose responses to the teaching situations/ vignettes show understanding of the use of the Ten Principles in the teaching	Increased acceptance of the teaching pedagogy/ methods based on the Ten Principles of the Thinking Mathematics Program • increased average value of acceptance of the items on the Linkert's scale which indicates instruction based on the Ten Principles • Increased % of trained teachers whose responses to the teaching situations/ vignettes show understanding of the Ten Principles in the teaching • Increased % of trained teachers whose responses show the application of the Ten Principles in the teaching		Linkert's scale Questionnaire for teachers Test for teachers with vignettes/teac-hing situations which measure pedagogical knowledge

Aim	Tasks	Activities	Baseline Indicators	Outcome Indicators	Impact Indicators	Methods/ instruments
			What is the level of the present mathematics' knowledge of teachers on the concepts that are introduced by the Thinking Mathematics • % of teachers whose responses to the tasks and to the teaching situations show knowledge and understanding of the concept of number, knowledge and understanding of the four arithmetical operations and their characteristics, and the knowledge and understanding in doing textual tasks and problem situations	Increased is the Mathematics' knowledge of teachers on the concepts that are introduced by the Thinking Mathematics • Increased is % of teachers whose responses to the tasks and to the teaching situations show knowledge and understanding of the concept of number, knowledge and understanding of the four arithmetical operations and their characteristics, and the knowledge and understanding in doing textual tasks and problem situations		Test for teachers that measure situations which measure mathematical knowledge
		Support to teachers in the implementat-ion of the Program		Acceptance, implementation and the need to support the Program • % на наставници кои се на определено ниво на примена на програмата • % на наставници кои се во определена категорија на загрижи за примена на програмата • % на наставници на кои им е потребна специфична поддршка		SBAM scale of concern for the implementati-on of the Program Self-assessment of teachers about the level of implementati-on Questionnaire for teachers

Aim	Tasks	Activities	Baseline Indicators	Outcome Indicators	Impact Indicators	Methods/ instruments
	School management gives support to qualitative instruction in the teaching of Mathematics in the grade teaching Increased state with teaching aids for the teaching of Mathematics	Support by the school management in giving support and in monitoring the introducing and the implementati-on of the Thinking Mathematics Program	Present level of support by the school management to the instruction of Mathematics in the grade teaching • % of representatives from the management that are not satisfied with the level of achievements in Mathematics in the grade teaching. • Approaches that are used in giving support to teachers in the teaching of Mathematics • The present state with teching aids for the instruction of Mathematics • Satisfaction of teachers for the support they have	Increased level of support by the school management to the instruction of Mathematics in the grade teaching • Increased % of representatives from the management who believe that their students could achieve more • Improved are the ways that are used to support the teachers in the instruction of Mathematics • Improved is the equipment of the school with teaching aids for the instruction of Mathematics • Increased is the level of satisfaction of teachers for the support they have		Semi-structured interview Questionnaire for teachers
Higher achievemen-ts of students	increased level of knowledge and understanding by students of the key mathematical concepts	The trained teachers are using the new approaches in the teaching	% of students who have correct responses on the tasks that measure knowledge and understanding of: -numbers -the four operations and their characteristics - doing textual tasks and problem situations		Increased % of students who have correct responses on the tasks that measure knowledge and understanding of: -numbers -the four operations and their characteristics - doing textual tasks and problem situations	Knowledge tests for students

SUPPLEMENT 2:

Pr/ Co	School 2009/2012	Urban/rural	Lan-guage	Change in 2012
Proj.	Hristo Uzunov	Ohrid	Mac	Josip Broz Tito, Struga
Cont.	Grigor Prlicev	Ohrid	Mac	
Proj.	Aco Sopov	Skopje	Mac	
Cont.	Zivko Brajkovski	Skopje	Mac	
Proj.	Dituria	Lipkovo – Kumanovo	Alb	
Cont.	Faik Konica	Slup-cane-Ku-manovo	Alb	
Proj.	Bratstvo-Edinstvo	Ohrid	Mac/ Alb	
Con-t.K	Braka Miladinovci	Struga	Mac/ Alb	
Proj.	Alija Avdovic	Batinci Sko-pje	Alb	Not includeduded and has no replace-ment
Cont.	Sami Fraseri	Skopje	Alb	Not included and has no replacement
Proj.	Kiril Pejcinovic	Tearce-Teto-vo	Mac/ Alb	The data for the instruction in Albanian language were collected in “Kiril Pejci-novic” Primary School – Tetovo; and for the instruction in Macedonian language in “Bratstvo-Edinstvo” Prim. School - Konjari
Cont.	Simce nastevski	Vratni-ca-Tetovo	Mac/ Alb	
Proj.	25-May	Skopje	Alb	
Cont.	7- Marsi	Skopje	Alb	
Proj.	Kiril i Metodij	Kocani	Mac	
Cont.	Nikola Karev	Kocani	Mac	
Proj.	11-October	Kumanovo	Mac	
Cont.	Braka Miladinovci	Kumanovo	Mac	
Proj.	Kiril and Metodij	Tetovo	Mac/ Alb	The data for the instruction in Albanian language were collected in “Kiril i Meto-dij” Primary School – Tetovo, and for the instruction in Macedonian language in “Bratstvo Migeni” Primary School - Tetovo
Cont.	Lirija	Tetovo	Mac/ Alb	
Proj.	Lazo Angelovski	Skopje	Mac	
Cont.	Gorgija Pulevski	Skopje	Mac	
Proj.	Sando Masev	Strumica	Mac	
Proj.	Dimkata A. Gaberot	Kavadarci	Mac	
Cont.	Tode Hadji Tefov	Kavadarci	Mac	
Proj.	Johan H. Pestalozi	Skopje	Mac	
Cont.	Kole Nedelkovski	Skopje	Mac	
Proj.	Jan A. Komenski	Skopje	Mac	
Cont.	Vera Ciriviri Trena	Skopje	Mac	
Cont.	Vidoe Podgorec	Strumica	Mac.	

SUPPLEMENT 3:

PSYCHOMETRIC CHARACTERISTICS OF THE INSTRUMENTS[19]

Scale of attitudes in learning Mathematics and in teaching Mathematics based on the Thinking Mathematics

The discriminativity of statements, item-test correlation ranges between 0.40 and 0.55 in 11 statements, and between 0.30 and 0.39 in 1, an in 2 statements it amounts – 0,25. This points out to a satisfactory discriminativity. The reliability is satisfactory (Cronbach alpha coefficient is 0.86 statements).

Test on pedagogical knowledge

The reliability is 0.55. The unsufficient reliability is due to the small number of requirements, and provided that the test had 40 requirements with similar characteristics, the estimated reliability would be 0.78.

The discriminativity (item-test correlation) is given in the table below. In the table, the number of requirements is given for each one of the categories: *very good requirement, good requirement, requirement that is satisfactory and poor requirement,* according to their discriminativity.

Category	Discriminativity	Number of requirements in the test
Very good	41 – 53	8
Good	31 – 40	2
Satisfactory	21 – 30	4

Test on Mathematics' knowledge for teachers

The reliability of the test is 0.74, and it is due to, also, the smaller number of requirements, and provided that the test had 40 requirements with similar characteristics, the estimated reliability of the test would be 0.77.

The table below gives the number of requirements in each of the categories: *very good requirement, good requirement, requirement that is satisfactory and poor requirement,* according to their discriminativity.

19 The psychometric characcteristics of the instrument were determined in the first measuring on a sample of 299 teachers and of 598 students.

Category	Discriminativity	Number of requirements in the teast
Very good	41 – 65	10
Good	31 – 40	13
Satisfactory	21 – 30	7
Poor	12 – 20	4

Scale of concern about the application of the approaches from Thinking Mathematics

An adapted 7-level scale of attitudes was used which contains 35 items, that have good measurable characteristics – as confirmed by numerious previous investigations[20].

Self - assessment of the levels of the principles and the approaches of the Project Thinking Mathematics

A question was used on self - assessmentof the level of application with a given description for each level. The instrument has good mesurable characteristics – as confirmed by numerious prevous investigations[21].

Test on Mathematics' knowledge for students

The test was checked on 597 students. On the basis of the results, the reliability of the conducted test is 0.79 and it is due to the smaller number of requirements. Provided that the test had 40 requirements with similar characteristics, the estimated reliability of the test would be 0.77.

The table below gives the number of requirements for each of the categories: *very good requirement, good requirement, requirement that is satisfactory and poor requirement,* according to their item-test correlation.

Category	Discriminativity	Number of requirements in the teast
Very good	41 – 65	14
Good	31 – 40	6
Satisfactory	21 – 30	1
Poor	< 20	0

20 Archie A George., Gene E. Hall., Suzanne M. Stiegelbauer (2008): Measuring implementation in Schools: *The Stages of Concern Questionaire*, SEDL (стр.11-23)

21 Archie A George., Gene E. Hall., Suzanne M. Stiegelbauer (2008): Measuring implementation in Schools: *Levels of Use,* SEDL, и Hall, G.E., Dirksen, D.J., and George, A.A. (2006). Measuring Implementation in Schools: Levels of Use. Austin, TX: Southwest Educational Development Laboratory (SEDL).

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